Interleaved Maths | Essential Connections 11

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How to use this book

Did you know most people are far more capable of maths than they realise? Our brains are basically designed to find patterns, and maths entirely formed from patterns. In this book we use patterns for Concepts, patterns for **Strategies** and patterns for Thinking Processes. The information below will help you know what to look for, why it's important, and how to put it together to really understand the patterns in maths.

Concepts

Every content area in maths is formed from a combination of just Five Key Concepts. If we don't know how to look for them, these Five Key Concepts can remain hidden for a long time. However, once we know what to look for, maths becomes far easier to understand and also a whole lot more fun. The first two chapters of this book help you to develop a solid understanding of these concepts so that you have a solid foundation to build on. You will also find a reference to the Five Key Concepts just beneath the title for every lesson to help cue you into the important foundational ideas. When new ideas are introduced, each will be clearly linked with one or more of the Five Key Concepts so that it builds on from what you already know. This also means that learning the new concept will seem much easier because you already know most of what you need to. The concepts that we will be using are:

- Partitioning: You can break a quantity into parts. Putting the parts back together again forms your original quantity.
- Relative Size: Numbers can be thought of as big or small only in relation to other numbers. This helps us to position numbers along a Number Line. For example, 100 is very big compared to 10, but very small compared to 1000. Our Place Value system uses multiples of 10 to space numbers out. 10 is ten times as big as 1. 100 is ten times as big as 10. 1 000 is ten times as big as 100.
- Multiplicative Thinking: Multiplication and Division can be pictured as Arrays or Area, like the top of a Lego block. Thinking in this way helps us solve a range of problems that a "groups of" model can't help with.
- Proportional Reasoning: Fractions, rates and ratios all make use of the relative size or proportion of the parts to the whole. They are multiplicative by nature (formed by multiplying or dividing). Proportional reasoning combines the concepts of partitioning, relative size and multiplicative thinking to reason about the size of the parts in rational numbers.
- Generalising: When we can find a pattern that always works we can generalise it into an equation or number sentence that always follows the same rules. Generalising allows us to program technology to do our calculations, which makes maths far more powerful.

Strategies

Some strategies are more important than others. Rather than memorising a whole bunch of formulas, this book makes use of **Flexible Strategies** that feel intuitive, are very efficient and help you to apply what you know in real world problems. Every one of the **Flexible Strategies** that we will look at are useful across different content areas. Every time you see the name of a **Strategy**, you can be assured that you will use it multiple times and in multiple chapters.

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Take your time to truly understand how each strategy works so that next time we use it you can build on what you already know.

Thinking Processes

Within each lesson you will use some of eight Thinking Processes. In every chapter you will find all eight processes, but not every process is in every lesson. The processes are always used in the same order because they help you to think through and make sense of maths. Here is a quick description to help you understand what kind of thinking to do for each process.

1. **Experiment**

   When you are given a new problem that you haven't seen before: have a guess (make a conjecture), brainstorm some ideas to try, and then try them out to see both what works and what doesn't work. While it sounds strange to try out your own ideas *before* being shown how to use the processes by your teacher, it sets your brain up to actually understand what you are being taught. You develop an intuitive sense of the concept.

2. **Explore**

   In the explore step we look for patterns, connections or similarities between the new learning and what you already know. Think about the Five Key Concepts and any **Strategies** that are suggested and use these to try out ideas for solving the problem.

3. **Evaluate**

   Think critically about what you have tried. What worked and what didn't work? Why did your strategies succeed or fail? What was it about them that was important? How could you adapt what you did to improve on it?

4. **Explain**

   Describe your process in a way that would make sense to someone else and try to make it as efficient as possible. This will make the process easier to understand later and help stick it in your brain.

5. **Practise**

   Practise your procedure until you feel confident with your ability to use it accurately and efficiently. Use the Interleaved Sets each week to recall a range of procedures.

6. **Apply**

   In this step you will take what you learned and apply it to a real-world context. Application questions force you to think a bit flexibly and to adapt what you have done rather than just sticking numbers into formulas. They also help you to think about maths beyond school.

7. **Extend**

   This step requires you to adapt or manipulate what you have done to solve a more complex question. Try to focus on altering the procedure, rather than starting from scratch.

8. **Investigate**

   This is a larger-scale real-world problem that requires you to apply skills you have been learning. Research the problem to find background information. Design a model that you think might work. Test out your model and determine what you need to change. Validate that your approach, verify your solution/s and list any limitations.

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Foundations of Place Value, Percentage and Proportions

This chapter sets the foundation of concepts and strategies that we will use throughout this book. It introduces flexible strategies for adding, subtracting, multiplying and dividing. It also helps us to understand fractions, decimals, percentages, rates and ratios. Take your time to truly understand the ideas in this chapter as the concepts and strategies will be used again in later lessons.

Learning Outcomes

The following skills from this chapter align with the Learning Outcomes for the Essential Mathematics subject.

	Adding and subtracting numbers using efficient strategies
	Multiplying and dividing numbers using efficient strategies
	Understanding which order to perform operations in, including when using a calculator for multi-step calculations
	Using leading digit approximation to estimate when performing operations
	Understanding how tenths, hundredths and thousandths work in decimal numbers, including rounding and approximating
	Determining if an answer is reasonable
	Connecting fractions with decimal and percentage representations
	Recalling decimal and percentage representations for commonly used fractions such as halves, quarters, fifths and tenths
	Calculating percentages: including percentage increases and decreases (discounts)
	Calculating rates and ratios

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Budget-Friendly Meal Planning Investigation

INVESTIGATION

When life gets busy people tend to eat more convenience foods, which are generally unhealthy and expensive. Getting organised by creating a list of recipes for most meals each week saves you time, stress, money and improves your diet.

Brief

You will work in a group to calculate the costs, health benefits and time required to cook several meals over the course of one week. You will use what you find out to create a set of three recipes that are healthy and budget-friendly, along with shopping lists of all ingredients for one of the scenarios below. You will adjust your recipes and shopping lists to fit the scenario of your choice.

Please note: Try to choose recipes that make use of similar ingredients to limit wastage and keep costs low. You may include recipes of your own design.

Scenarios

Different living situations have a big impact on our meal choices. You will need to provide three meal recommendations to feed people in the following scenarios for one week of dinners:

Skills Checklist

Skills you need to demonstrate for this assignment:

- o Calculate, measure, estimate and convert units of mass, volume and capacity
- o Use, convert and calculate units of energy including calories and kilojoules
- o Add, subtract, multiply and divide numbers
- o Use rates and ratios in practical situations
- o Apply percentages, including discounts
- o Calculate best buys
- o Round numbers and apply approximation strategies
- o Substitute values into expressions

1. A single person, who only wants to cook three times in a week then eat leftovers for the other three meals. You need food for six meals for one person.
2. Three housemates, who each cook once in a week with leftovers for the other three meals. You need food for six meals for three people.
3. A family of five, cooking three times in a week and eating leftovers for the other three meals. You need food for six meals for five people.

Hand in

- o Your 3 adjusted recipes, making sure that they feed the correct number of people.
- o An ingredients list for shopping.
- o The total cost of each recipe, and the cost per serving.
- o A description of the number of kilojoules per serving, as well as the fat and carbohydrate content per serving.

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1. Relative size of numbers to 1000

RELATIVE SIZE

Relative size underpins concepts such as fractions, percent, converting measurements and even probability. It is a fundamental idea that we will use to build many other concepts.

Experiment

Copy the number line beneath on a large piece of paper. Where would these numbers fit?
10, 100, 200

0 1000

Does your first idea look right? Try adding in all the other hundreds and consider if the spacing is reasonable. It is not unusual to try 10 different lines before the spacing works.
This Number Line strategy will be used frequently in later lessons and chapters.

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Explore

Stimulus questions

- How many hundreds make up 1000?
- How many tens make up 100?
- Where would 110 go? 120? 130?
- How about 210? 220? 230?

Patterns and connections

What do you think should go in the exact middle of the line? How come? Redraw your line below your original line with this in mind (only if you need to change your mind).

Evaluate

What key numbers or position points can be used to help you figure out positions on a number line from 0 to 1000? If you changed your mind, what did you change your mind about?

Explain

There is only one correct position for each number.

- How do you know that your numbers are in the right place?
- What numbers or positions would you start with next time?

Extend

- Use a strip of paper one metre long. Create your number line on this strip, making sure that the scale is correct by folding the paper into the correct size partitions. You will need this for later lessons.

Work with a friend on at least two of the tasks below:

- What would happen if your number line scale changed to 0-100, 0-10 or 0-1 on the original line? What key numbers or position points would be used to help position other numbers?
- What would change if your number line scale was to 0-2000? What key numbers or position points would be helpful?
- What if your number line showed a bank balance where you could borrow up to $1000? How could you apply similar thinking to show negative balances?

Apply relative size in real life situations

Set A: Length

Use the one metre long **Number Line** that you made to measure the length and width of the following items. Notice that 0-1000 is the distance in millimetres. A metre-long line from 0-100 would show centimetres.

1. Your desk
2. Your classroom
3. Your pen or pencil

Set B: Convert units of length

Use your one metre long number line to consider the connections between the following amounts. Complete the following table to show conversions for units. You don't need to follow any rules however you should try to find connections and to explain any that you notice.

Metres	Centimetres	Millimetres	What connections can you see?
	7cm		
		7mm	
7m			
	75cm		
		75mm	

The strategy shown below is called using a **Place Value Chart**. It will help you with conversions. **This strategy will be used frequently in later chapters.**

Thousands (of ones)	Hundreds (of ones)	Tens (of ones)	Ones.	Tenths (of ones)	Hundredths (of ones)	Thousandths (of ones)
kilo			Base unit (m)		centi	milli

Set C: Integers, negative numbers and money

A **Number Line** visually representing a bank balance between -100 to +100, with zero in the middle, is a great way to visualise negative numbers. The negative end represents owing money, such as spending with a credit card. Sketch the line and use it to work out the answers to the following questions.

1. I have $80 in the bank.
2. I use a credit card and spend $100. Draw an arrow to show what happens on the line and mark the end position.
3. I spend another $20. Draw an arrow to show what happens and mark the end position.
4. I deposit (add in) $50 into my bank account. Draw an arrow to show what happens and mark the end position. What is my bank balance?

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2. Efficient addition strategies

Partitioning

Many addition strategies that we were taught have been inherited from times before technology was around to help with complex situations. This lesson is designed to teach more efficient strategies for simple addition situations, starting with the largest place. These strategies are also great for approximation and easy to carry out mentally. They will be used in other chapters.

Explore

Vertical format, largest place first

Instead of adding from the right, add from the left and write a number on each line below.
Use the worked example to understand how the strategy works, then try the questions set.

Worked example

$$
\begin{array}{cccc}
 & 2 & 7 & 3 \\
+ & 3 & 6 & 5 \\
\hline
 & 5 & 0 & 0 \\
 & 1 & 3 & 0 \\
 & & & 8 \\
\hline
 & 6 & 3 & 8
\end{array}
$$

1. 347 + 492
2. 37 + 94
3. 2508 + 3892
4. 347 + 492 + 156
5. 47 + 492
6. 237 + 94
7. 508 + 3892
8. 347 + 92 + 156

Sample

Horizontal Format Flexible Strategy

This strategy is very similar to the first but involves a different layout. It is particularly useful as you can also use it for negative numbers, fractions and decimals. Use the worked example to understand how the strategy works, then try the questions set. They use the same numbers as Set B above. **This Horizontal Format strategy will be used in later lessons.**

9. 47 + 492
10. 237 + 94
11. 508 + 3892
12. 347 + 92 + 156

Worked example

273 + 365
Let's think hundreds, tens, ones
= 500 + 130 + 8
= 638

Evaluate

1. What did you like about these strategies? Which did you prefer? How do you think starting with the largest numbers might be useful for mental calculations rather than written calculations?
2. Is there a way that you could use just your first calculation to come up with a reasonable range for possible answers without calculating the whole lot? Think: "It will be at least… and not more than…". **This strategy is called Leading-Digit Approximation and will be used in later lessons.**

3. Relative size of negative numbers

RELATIVE SIZE

Negative numbers are very important in finance. We often use credit cards and loans, so need to understand what happens with loan balances when we deposit (add money in) or withdraw/spend (take money out). This concept was introduced in Set C from Relative size to 100.

Experiment

Apply the **Number Line Strategy**. Where would the following numbers fit on the scale below? Try using a very large piece of paper.

-50, 25, 0, -75, 50, 75

-100 +100

Explore

Make strips of paper that are 50, 25 and 75 long when compared to your number line. Using those strips of paper and your number line, write as many number sentences as you can using just those numbers.

Stimulus questions

Write equations for these situations and answer them:

- Start at 75, end at 25: what happened?
- Start at 25, end at 75: what happened?
- Start at 25, end at -50: what happened?
- Start at -50, end at 25: what happened?

Patterns and connections

Find the connections between the four stimulus questions above.

Evaluate

Use your strips of paper, the number sentences you came up with in the Explore section, and the patterns you found to evaluate the following:

- How is 75-25 similar to 25-75? Why?
- Would this same pattern apply to other numbers? Try 8-3 and 3-8: what happens? Try 17-3 and 3-17: what happens?
- Would this pattern always be true or only sometimes?

Explain

If this pattern is always true, how could you use it to subtract larger numbers from smaller numbers?

Extend

Work with a friend to draw a line from -2 to +2.
Place the following decimal numbers on the line: 0.5, 1.5, -0.5, -1.5, 0.25, -0.25, 1.25, -1.25

Write a hard question that someone who had good understanding of negative numbers could answer. What would you want to see in their answer?

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4. Efficient subtraction strategies

PARTITIONING   RELATIVE SIZE

Understanding negative numbers means that we can subtract using much more efficient methods. Instead of starting with the ones, the following methods start with the largest numbers. These methods are simpler to perform mentally and make approximations easier as well.

Strategy one: start with the largest place, vertical format

Instead of subtracting from the right, start from the left and write a number on each line below. Use the worked example to understand how the strategy works, then try the questions set.

Worked example

	3	6	2
-	1	7	5
	2	0	0
-		1	0
-			3
	1	8	7

Set A:

1. 347 - 192
2. 94 - 39
3. 2508 - 1892
4. 2001 - 901

Set B:

5. 407 - 49
6. 230 - 94
7. 49 - 407
8. 94 - 230

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Strategy two: Horizontal Format

Apply the Horizontal Format strategy from lesson 2. Use the worked example to understand how the strategy works, then try the questions set. They use the same numbers as Set B above. **This strategy will be used in later lessons.**

Worked example

362 - 175
= 200 -10 -3
= 187

Set C:

9. 407 - 49
10. 230 - 94
11. 49 - 407
12. 94 - 230

You could also use a **Number Line** strategy and jump along the line to calculate the answers.

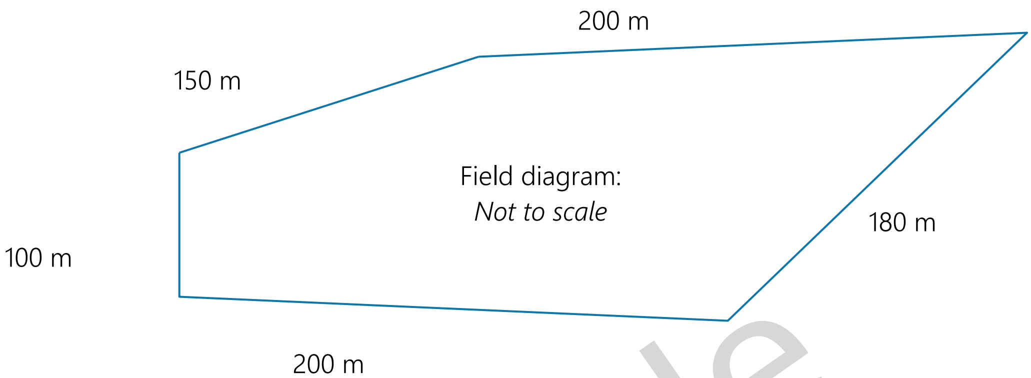
Reflect

1. What did you like about these strategies? Which did you prefer?
2. How do you think starting with the largest numbers might be useful for mental calculations rather than written calculations?
3. Look at your answers for each set and the first calculation that you did. Is there a way that you could use just your first calculation to come up with a reasonable estimate without calculating the whole lot? Think: "It will be at least... and not more than...". This is called "leading-digit approximation".

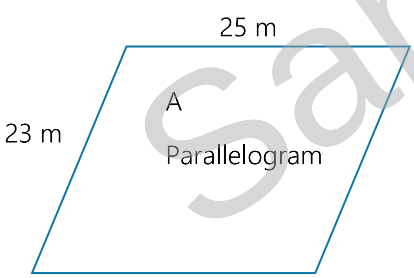
5. Application of addition and relative size to perimeter

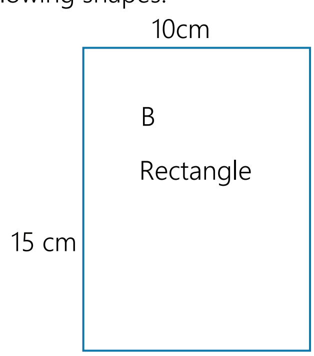
PARTITIONING RELATIVE SIZE

Perimeter is one of the most common calculations used in building, renovating and allocating land. You don't need a formula, but you do need to apply your understanding of what we have been learning in this chapter to work it out. Use the following diagram to deduce what perimeter means and how to figure it out. Think about using your Number Line strategy to help.



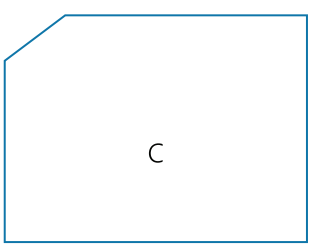
1. The perimeter of the field in the diagram above is 830m. How do we calculate perimeter?
2. Use the same method to work out the perimeter for the following shapes:





Set A: Use your **Number Line** metre strip to work out perimeter of the following

3. Your desk
4. Your maths book
5. A square that has one side of 7cm
6. A rectangle with a length of 12cm and a width of 8cm
7. These shapes:



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6. Relative size with decimal numbers

Relative Size **Proportional Reasoning**

Understanding relative size with length helps us to unpack decimal numbers. The decimal point in a number only tells us where the ones are. Discuss the following stimulus questions, then use what you unpack to experiment with the problem below. This is the ***Of A Dollar*** *strategy.*

Discussion questions

1. In our number system we work on a base number. This number is used when we have to move from ones to tens, or from tens to hundreds, or from hundreds to thousands because we have too many to fit in that place. What number is this?
2. Why do you think we chose 10 to be the basis for our number system? Hint: it has to do with our bodies.
3. Each time we move one digit to the left in our base ten system, the number is ten times larger. For example, 100 is ten times larger than 10. Likewise, each time we move one digit to the right, the number is ten times smaller. For example, 50 is ten times smaller than 500.
   The same is true with decimal numbers:
   - 0.1 is ten times bigger than 0.01
   - 0.002 is ten times smaller than 0.02

Relative Size
Proportional Reasoning
"Deci" is Latin for "tenth". Decimal numbers are therefore "base ten". Money is also "base ten", so we can use it to understand converting fractions to decimals.

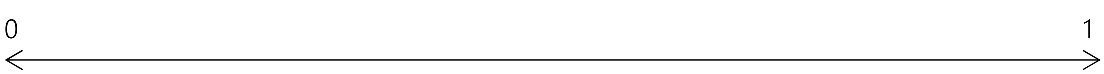
What is half **Of A Dollar**? Write that in dollars and cents. That is how we write ½ as a decimal number.

What is one quarter **Of A Dollar**? Write that in dollars and cents. That is how we write ¼ as a decimal number.

Sample

Experiment

Where would the following numbers fit on a scale of 0 to 1? Draw the line below on a large piece of paper. Use the strategy **Of A Dollar** to convert the following fractions into decimals and write them on the line: one tenth, two tenths, one half, one quarter, one fifth, two fifths



Explore

Does your first idea look right? Try adding in all the other tenths and consider if the spacing is reasonable.

- Where would 50 hundredths fit?
- What do you notice about its position?
- What other decimal numbers and fractions are at the same position?

Evaluate and Explain

What key numbers or position points can be used to help you figure out positions on a number line from 0 to 1?
With whole numbers adding a zero to the right shifts all digits to the left, making the number ten times bigger. Why doesn't adding a zero to the right of a decimal number do the same thing and make it ten times bigger?

7. Apply "Relative Size" with decimals to estimation

RELATIVE SIZE PROPORTIONAL REASONING

*When we understand the relative size of decimal numbers, it is easier to estimate and to figure out what a reasonable calculation would be for lengths. Your **Number Line** strategy will help.*

Set A: Estimation and approximation

Use a strip of paper that is one metre long as a line from 0 to 1. Estimate where the following decimal numbers would be. Consider what decimal number is close to the amount you are given and what fraction is roughly the same. Mark them on your line and explain how you worked out where each should be placed.

1. 0.496 Close to: Nearby fraction:
2. 0.102 Close to: Nearby fraction:
3. 0.253 Close to: Nearby fraction:
4. 0.989 Close to: Nearby fraction:
5. 0.81 Close to: Nearby fraction:

Set B: Ordering and comparing lengths

Compare the following sets of measurements. Use the box on the right to help you to convert them into metres so that you can compare them.

Rewrite each set in ascending order (smallest to largest), including explaining if any numbers are equal.

6. 0.5m, 0.50m, 0.05m
7. 49mm, 4.9m, 49cm
8. 2.5cm, 250mm, 0.0025m
9. 0.5m, 5cm, 0.50cm

RELATIVE SIZE

When converting between measurements it is helpful to think about whether you need more or less.

For example, consider 1m. There are *more* cm in a m, because cm are smaller so more fit into the same space. That means that after converting from metres into to centimetres, you should have *a larger number*.

m to cm: *you should have more*
1m = 1 (sets of 100) = 100cm
2m = 2 (sets of 100) = 200cm

cm to m: *you should have less*
500cm = 500 (split into sets of 100) = 5m
250cm = 250 (split into sets of 100) = 2.5m

Set C: Rounding decimal numbers

In Set A above you considered what decimal number was close to the one you were given. This is an informal way of rounding. When we are rounding numbers, we consider which number on either side is closest to our amount. For example, if rounding 0.463 to the nearest one we would consider if it was closer to 0 or 1; if rounding it to the nearest tenth we would consider if it was closer to 0.4 or 0.5; if rounding it to the nearest hundredth we would consider if it was closer to 0.46 or 0.47. Complete the table to round the following numbers.

Original number	Nearest one	Nearest tenth	Nearest hundredth
0.463			
0.555			
0.494			
0.209			

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8. Connecting fractions, decimals and percentage

RELATIVE SIZE   PROPORTIONAL REASONING

*Fractions, decimal numbers and percentages are different ways of representing the same amount. When we looked at Relative Size of decimal numbers, we used the strategy **Of A Dollar** to help us turn fractions into decimals without requiring a formal process. In this task we will explore further connections between money, measurement, decimals, percentage and fractions to help solidify connections and make converting between them easy.*

Explore

- How many **cent**s are in each dollar?
- How many **centi**metres in each metre?
- How many years in each **cent**ury?
- What number do you think the prefix **cent** represents?
- What do you think per**cent** means?
- What per**cent** is equal to one whole?

Find the pattern:

Consider half **Of A Dollar**:

How many cents in half a dollar?
Write the number of cents, and also write the amount in dollars.

- Write a half as a decimal number.
- What is half written as a percent?
- What pattern can you find?

Consider one quarter **Of A Dollar**:

How many cents in one quarter of a dollar?
Write the number of cents, and also write the amount in dollars.

- Write a quarter as a decimal number.
- What do you think one quarter might be when written as a percentage?
- What pattern can you find?

Consider one fifth **Of A Dollar**:

Write the number of cents, and also write the amount in dollars.

- Write a fifth as a decimal number.
- What do you think one fifth might be when written as a percentage?
- What pattern can you find?

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Extend and Experiment

One particular operation will help you to convert fractions into decimals and therefore, into percentages. Use what you have worked out in the exploration section above to experiment with a calculator and find out which operation to use to turn your fractions into decimals.

For example, to turn ½ into 0.5 you need to press these keys: 1 ___ 2 =
Work out what operation goes on the line.

Use either the **Of A Dollar** idea or the operation you found above to convert the following fractions into both decimals and percentages, then place them on a **Number Line**:

$\frac{1}{5}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{2}{5}$ $\frac{2}{4}$ $\frac{2}{3}$ $\frac{3}{5}$ $\frac{3}{4}$

0 1

9. Apply fractions, decimals and percentage in the real world

RELATIVE SIZE PROPORTIONAL REASONING

Work with a partner to apply your understanding of converting between the fractions, decimals and percentages to the following problem sets. Try to work out each answer mentally first, then use a calculator to check that you are on the right track.

Set A: Length of fabric

Fabric is purchased in centimetres. Convert each of the following amounts into centimetres.

1. Two and a half metres ($2\frac{1}{2}$ as a mixed number)
2. One and a quarter metres ($1\frac{1}{4}$ as a mixed number)
3. Window frame measures 2750mm: write it in cm for fabric, in metres so that we can easily picture it, and as a mixed number for simple comparison of size.

Set B: Discounts

Discounts are often expressed as percentages. It is important to be able to think about how big each discount is. Express each of the discounts below as a fraction. The first is done for you as an example. **This strategy is called Partition It and you will use it frequently.**

Discount percentage	Fraction (Of A Dollar strategy)	Partition It: to work out what is left (you pay):
25% off 25c per dollar off	$\frac{1}{4}$ off	75% of the original price, or $\frac{3}{4}$ of the original price
20% off		
10% off		
		60% of the original price, or $\frac{6}{10}$ of the original price
	$\frac{4}{5}$ off	

Set C: Chance of events occurring

Chance or probability is expressed as a likelihood between 0 (will never happen) and 1 (will always happen). Work out the fraction, decimal and percentage for each of the following situations and place it on the number line from last lesson. Use a **Number Line** if it helps.

1. Flipping a coin and having it land on heads
2. Choosing a spade from a deck of cards
3. Rolling a 3 on a die
4. The sun rising tomorrow
5. The sun rising again today
6. Choosing the correct number between 1 and 10

Sample

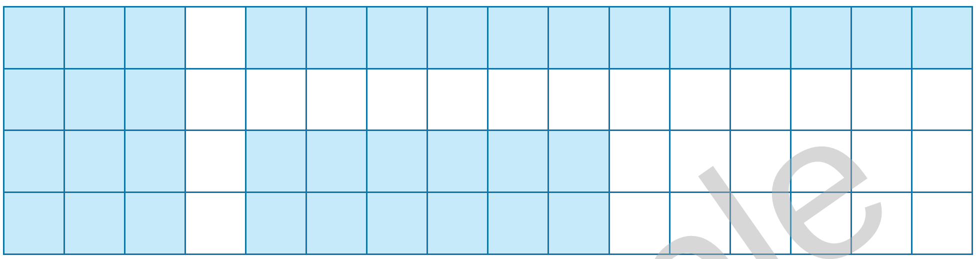
10. Multiplication with arrays

MULTIPLICATIVE THINKING

Multiplicative thinking underpins concepts such as area and volume and helps us to link fractions, decimals and percent. It is a fundamental idea that we will use to build other concepts.

Experiment

The diagrams below show rectangles or arrays formed from 12 squares drawn on grids. Use these diagrams to stimulate your own thinking and come up with as many different rectangles as you can formed from 36 squares. This is called the **Array Strategy** and it will be used frequently.



Sample

Explore

Each array that you have drawn represents a multiplication fact. Write the multiplication fact represented next to each array.

6x6 makes a square. Do you think this is still a rectangle? Discuss this with your group and check your thinking with your teacher.
What arrays do you think you could make with 35 squares? How about with 37 squares?

Evaluate

Imagine that you turned each array by a quarter. What other multiplication facts are now represented? Do you think that this would work with all multiplication facts, or just these ones? Explain your thinking.
Find the rectangle of 12 above that is 3x4.

Both 3 and 4 are **factors** of 12. 12 is the **multiple** of 3 and 4. What factors can you find for 36? **Use An Array.**

Draw as many arrays as you can with 7 squares. Now try 11 squares. What do you notice? 7 and 11 are both **prime** numbers whereas 12 and 36 are **composite**. What do you think square numbers might look like?

Explain

Using the questions above to stimulate your thinking, describe the following terms in your own words. Include a diagram if it is useful.

- Factor
- Multiple
- Prime
- Composite

Extend

The **Array Strategy** for multiplication can help us to multiply much larger numbers. Sketch an array to show 23 x 45. How you could break your large rectangle into smaller parts to make multiplication easier? Draw the parts and show your thinking.

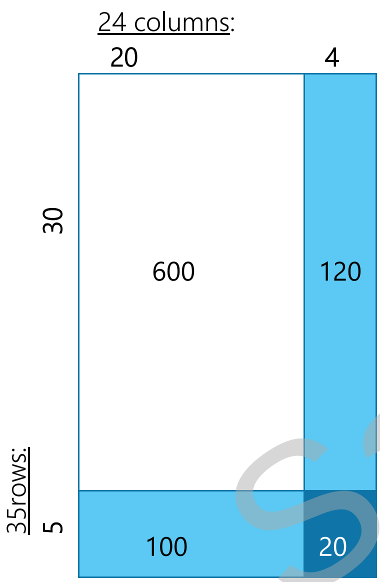
11. Efficient strategies for multiplication

MULTIPLICATIVE THINKING   PARTITIONING

Many multiplication strategies that we were taught involve memorising a procedure then trying not to mess it up. This lesson is designed to teach more efficient strategies for multiplication, starting with the ***Array Strategy****. This strategy is also great for approximation and easy to carry out mentally.*

Strategy one: sketch an array and then partition

The following diagram shows an array of 24 x 35. What do you think you might do with each part to work out the answer? The answer is 840. Use this to work out how to use the strategy.



Set A: Different numbers of digits

Try your strategy out on the following questions. Your sketches do not need to be to scale.

1. 27 x 5
2. 45 x 34
3. 6 x 213

Set B: Whole numbers by decimal numbers

The same strategy of an array can help you to multiply decimal numbers easily. Remember if you get stuck to think of the decimal numbers as money.

4. 1.5 x 4
5. 2.2 x 6

Sample

Strategy two: partitioning arrays for difficult to remember facts

Sometimes, particular multiplication facts are easier to remember than others. Most students find 7s particularly difficult to memorise, and 7s do not have an easy counting pattern if you get stuck. Luckily, the strategy shown above for breaking arrays into parts helps with 7s.

Sketch an array of 7x6. Now consider how you could partition the 7 to make easier parts. Draw it, show each part, and discuss how you cut the array with your friends. Did you do it the same way? What was easiest for you? How might you apply this same thinking to multiply a number by 12?

Strategy three: finger patterns for 6, 7, 8 and 9 facts

One final strategy is to use your fingers to quickly calculate 6s, 7s, 8s and 9s. This strategy was used in the middle ages for quick calculations. Your teacher will demonstrate the strategy. Discuss it with a partner and reflect on which of the strategies you think will be useful.

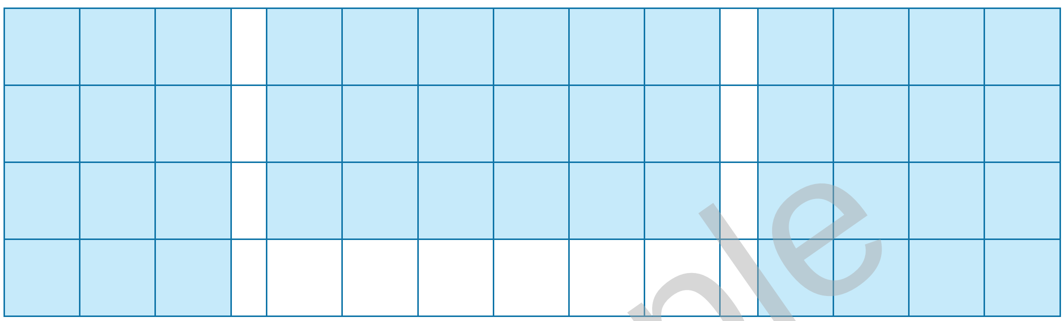
12. Application of arrays to area

MULTIPLICATIVE THINKING

*Area is one of the most common calculations used in building, farming, renovating and town planning. You don't need a formula, but you do need to apply your understanding of arrays to work it out. Use the following diagram to deduce what area means and determine how to calculate the area of a rectangle. This lesson applies the **Array Strategy**.*

Experiment

Count each of the rectangles below to determine how many squares make up the area.



Sample

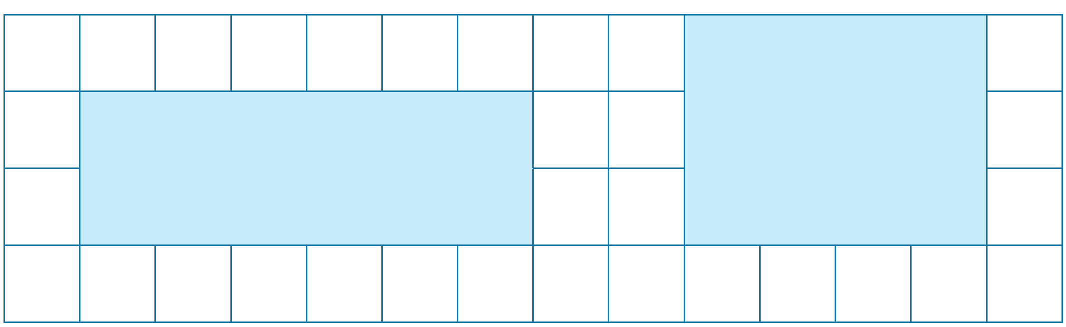
Using your knowledge of arrays, what would be another way that you could determine the area of each without having to count every square?

Explore

Find the rectangle shown above that has a base of 6cm and a height of 3cm. How could you use the base and height measurements to determine the area in cm^2? Try your idea with the other rectangles shown (including the square as it is a special type of rectangle). What do you notice? How can you write your idea as a formula using the variables Area (A), Base (B) and Height (H)?

Evaluate

Is there a way that you could use your formula if you didn't know how many squares were in the array? Try it on the following rectangles.



Explain

Explain how you would use your formula and your ideas so far to calculate the area of the following rectangles:

1. Your desk in cm^2
2. Your classroom floor in m^2
3. A rectangle that was twice as long as it was high. The height was 5cm.
4. A square that was 6cm high
5. A square that was half a metre long – how many square metres does it take up altogether?

Extend

To measure area, we use square centimetres (cm^2), square metres (m^2) and hectares. These units increase in size at the same rate.

Use this idea to work out:

1. How many square centimetres fit in one square metre?
2. How many square metres fit in one hectare?
3. What would a hectare look like if it was a square?

Sample

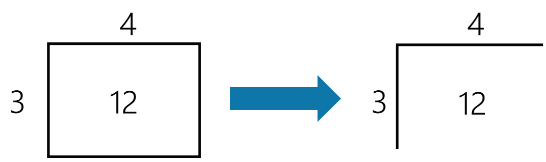
13. Division with arrays

MULTIPLICATIVE THINKING

*Multiplicative thinking underpins division, and forms a very strong link to fractions, decimals and percentage. In this activity, you will link your **Array Strategy** to division.*

Previously, we examined arrays made with 12 and with 36 squares. Division can be thought of as making arrays when you know the total amount of squares and the number of rows, and you have to find the missing number of columns (or vice versa).

Using this thinking, an array with 3 rows of 4 to make 12 can look like the first picture below. It also looks similar to the symbolic representation for short division shown in the second picture.



Experiment

In this activity, we will apply thinking about using the **Array Strategy** with grids of squares to dividing when the divisor is not a factor of the dividend. That means you will need to think about what to do with the left-over squares that do not fit. Use grid paper to draw a rectangle that has a total area of 12 squares, but with exactly 5 columns (5 squares are along the top of the rectangle). We can also write this as 12 ÷ 5.

Sample

Explore

While you could leave the left-over squares as remainders, try instead to cut them into equal parts and spread the parts out between the columns. What have you tried? What has worked or not worked?
You should be able to find at least two answers that use fractions of the square.

What do you notice about the fraction you now have and the numbers in the original question? How do you think fractions and division might be connected?

Try thinking about dividing $12 between 5 people. How much money would each person get? Write it as a decimal number.

Evaluate

Fraction answer: If you have ended up with a mixed number (whole number and a fraction) as your answer, try turning it into an improper fraction.

Explain

Share all your answers as a group. Try to find connections between each answer.
What patterns can you find?

Extend

Try to draw an array made of 10, with sides of only 3. What happens? What do you notice?

14. Division, fractions and decimals

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

Work with a partner to look at the worked example, then apply your understanding of the connections between division, fractions and decimals to the following problem sets. Try to work out each answer mentally or by drawing first, then use a calculator to check. Division, fractions, decimals and percentages are all representations of the exact same amounts.

Strategies:
Array Strategy gives you a fraction answer.
Of A Dollar Strategy gives you a decimal answer.

MULTIPLICATIVE THINKING
PROPORTIONAL REASONING

17 ÷ 4 can be shown in different ways. All the following methods represent the same amount.

$$\begin{array}{r} 4\ \text{r}1 \\ 4\overline{)\,1\ \ 7} \end{array}$$

$$\begin{array}{r} 4\frac{1}{4} \\ 4\overline{)\,1\ \ 7} \end{array}$$

$$\begin{array}{r} 4.\ 2\,5 \\ 4\overline{)\,1\ \ 7.\ {}^{1}0\ {}^{2}0} \end{array}$$

Set A:
Give each answer as a remainder, fraction and decimal
1. 6 ÷ 4
2. 6 ÷ 5
3. 9 ÷ 4
4. 9 ÷ 5
5. 29 ÷ 2
6. 29 ÷ 3
7. 29 ÷ 5

Set B:
There are times when the way you express a remainder matters. For each of the situations below decide which remainder is most appropriate and explain why.
8. Making teams of people from a class
9. Sharing cups of flour between bowls
10. Allocating money to budgets
11. Sharing cupcakes between friends
12. Sharing wrapped lollies between friends
13. Allocating time to different activities when there are not whole hours involved
14. Sharing a total land area among potential allotments
15. Sharing prize money between a team

Sample

Extend
Sometimes, people think that small numbers cannot be divided by larger numbers. This isn't true because we can split the small number into fractions or decimals. Try drawing an array to show a total area of 2 squares, with a length of 3 squares. This is the same as 2 ÷ 3. You can think of it as 2 cakes shared between 3 people, or $2 shared between 3 people if that is helpful. Draw the diagram and explain your thinking. Discuss what you have done with a friend.

15. Operations with multiple steps

GENERALISING

Work with a partner to look at the worked example, then figure out which of the orders given below for operations is correct. The correct answer will obey all four rules in the worked example. You may use a calculator as needed.

GENERALISING

First rule:
$4 \times 5 \times (9 + 3) = 240$

Second rule:
$3 + (2+3)^2 = 28$

Second rule:
$4 + 5 + 3 \times 6 = 27$
$4 + 5 + 15 \div 3 = 14$

Third rule:
$5 \times 6 \div 2 \times 3 = 45$

Fourth rule:
$6 - 3 + 4 - 5 = 2$

Evaluate

Which **one** of the following statements gives the correct order for operations?

A. Brackets, indices, multiplication, then division, then addition, then subtraction
B. Brackets, addition and subtraction as they appear, then multiplication and division as they appear
C. Brackets, indices, then work left to right with whatever operation appears
D. Brackets, indices, multiplication and division as they appear, then addition and subtraction as they appear

Set A:

Calculate the solutions for the following equations. You may use a calculator, but you need to show your working as well.

1. $7 \times 9 + (3 + 7) =$
2. $4 + 3 - 2^2 \times 3 =$
3. $(5 - 3)^2 \times 5 + 9 =$
4. $12 \times (3 + 2) \div 10 =$
5. $6 \times 7 \times 2 \div 12 =$
6. $12 \div 2^2 \times 5 =$
7. $12 + 14 \div 2 =$
8. $19 - 5 \times (7 - 4) =$
9. $7 - 4 + 7 - 3 - 1 =$
10. $12 + 4 \times 5 \div 2 - 11 =$

Set B:

Write equations for the following situations. Make sure that you use correct order of operations. Solve each equation and show your working.

11. Jacki had three prepaid SIM cards worth $25 each. She spent $67 on phone calls. How much does she have left to spend?
12. Michael had 12 chocolates which he divided into 3 bags. Then he added another 5 chocolates to one of the bags. How many chocolates were in that bag?
13. The price of a shirt was marked at $5 off from the original price of $20. Janessa bought 6 shirts. How much did she spend?

Set C:

Write a situation for each equation. You do not need to solve them.

14. $(60 + 3) \div 7$
15. $35 \div 5 - 4$
16. $7 + (8 \times 3) - 20$

Sample

16. Approximation using leading digits

GENERALISING

*Being able to approximate is incredibly useful. It allows you to quickly determine whether an answer is reasonable, for example, when considering a quote for work from a tradesperson. One of the easiest ways to approximate is by rounding all numbers to the leading digit, then performing the operations. Use the worked example below to figure out **Leading Digit Approximation** works. This strategy links with previous learning from lessons 2 and 4.*

Set A: Whole numbers

1. 345 x 920
2. 354 + 290
3. 354 – 902
4. 345 ÷ 920

Set B: Decimal numbers

5. 3.45 x 92.0
6. 35.4 + 2.90
7. 354 – 9.02
8. 34.5 ÷ 9.20

Set C: Multi-step equations

9. 35 x 980 + 0.87
10. 354 + 290 x 0.87
11. 35^2 – 902 + 78
12. 345 ÷ (920 ÷ 3.29)

Worked examples

23 432 x 558
≅ 20 000 x 600
≅ 12 000 000

2.95 x 0.845
≅ 3 x 0.8
≅ 2.4

967 ÷ 18
≅ 1000 ÷ 20
≅ 50

33.4 x 1.9 – 4.45
≅ 30 x 2 – 4
≅ 56

Sample

Set D: Application – approximation and reasonableness

Sometimes Leading Digit Approximation method is not appropriate for the situation as it results in an unreasonable answer. Complete the following approximations and comment on the reasonableness of the answers.

13. The Adelaide oval as a capacity of 53 500 people. It is approximately 95% full. How many people are there?
14. The mass of a ute is 1 837kg. The mass of its load is 930kg. How much does it weigh?
15. In 2019, 15 229 Australians were diagnosed with melanoma. Chance of surviving at least five years is 91%. Approximately how many will be still alive in 2024?
16. According to the Centre for Disease Control and Prevention, 36 000 Americans are killed by guns each year. What is the average number of people killed by guns per day?

Evaluate

What did you like about this strategy? What didn't you like?
When do you think this strategy would be useful?
When would you use something else instead?

17. Percentage of an amount

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

Percentages are one of the most commonly used mathematical concepts outside of a school context. In this activity, you will work with a friend to develop your understanding of percentages, then apply them to answer the questions below.

Stimulus questions: finding a percentage of an amount

10% of 4 = 0.4

- How do we write 10% as a decimal number?
- What operation does "of" usually mean? For example, 2 packets of 10 lollies.
- Use your calculator to determine how to find 10% of 4. The answer should be 0.4.

75% of 20 = 15

- How do we write 75% as a decimal number?
- Use your calculator to determine how to find 75% of 20.
- Alternatively, you could think about 75% as being the same as ¾ and work from there.

Stimulus questions: finding what percentage of an amount another amount makes up

15 out of 20 is 75%.

- How do we write 15 out of 20 as a fraction?
- What operation does "out of" usually mean? For example, 2 out of the 10 lollies were green.
- How do we write that fraction as a decimal, then a percent? Use your calculator to determine how to use the numbers 15 and 20 to get 75% or 0.75.

10 out of 40 is 25%.

- How do we write 10 out of 40 as a fraction?
- What operation does "out of" usually mean?
- How do we write that fraction as a decimal, then a percent? Use your calculator to determine how to use the numbers 10 and 40 to get 25% or 0.25.

Set A: Mixed practice and application

1. Find 70% of 40
2. What percentage is 45 out of 50?
3. Jen scored 80% on her exam. There were 20 questions. How many did she get correct?
4. Jen scored 12 out of 20 on her exam. What was her percentage correct? What was her percentage incorrect?
5. A shirt was marked down from $30 to $24. What percentage was the discount?
6. The interest rate for a bank loan was 4% per year. The loan was for $200 000. How much interest will be charged for one year?
7. A couple paid $15 000 interest on their loan of $300 000 last year. What was the interest rate as a percentage?
8. The superannuation rate is 9.5%. An employee earns $45 000 per year plus the 9.5% superannuation. How much superannuation is paid?

Sample

18. Percentage off

PARTITIONING PROPORTIONAL REASONING

Most people use quite complicated procedures when finding how much money they will pay for a discounted amount. This activity will teach you a much simpler way of thinking about discounts.

Stimulus questions: finding a percentage *off* an amount

25% off $20 = $15

- We could try finding 25% of $20 (which is $5), then taking that amount away from the $20.
- An alternative is to use our **Partition It** to work out the percentage that would be left. If we took 25% away from 100%, *what percent is left?*
- Try finding that percent of $20 using your calculator or a mental strategy.
- What do you notice?

20% off $30 = $24

- We could try finding 20% of $30, then take that amount away from the $30.
- An alternative is use our **Partition It** to work out what would be left. If we took 20% away from 100%, *what percent is left?*
- Try finding that percent of $30 using your calculator or a mental strategy.
- What do you notice?

Sample

Set A: Quick practice

1. Find 70% off $40
2. Find 30% off $60
3. What percentage discount was given if a shirt now costs $45, reduced from $50?
4. What percentage discount was given if a shirt is $10 off the initial price of $80?

Set B: Complex Application

The following questions have a mix of percentage *of* and percentage *off*, with multiple steps and some that require you to work backwards. Diagrams will help. Work with a friend if you find it helpful. Use the **Partition It** strategy if it helps.

5. Hannah earned 20% of the total tips and Adam earned 30% of the tips. Adam was paid $45 for his 30%. How much was Hannah paid? What was the total amount tipped over the course of the shift?
6. Laura bought 3 shirts for 50% off and paid a total of $37.50. What was the original price of each shirt?
7. 25% of the children in the class were home with chicken pox. Another 15% were at choir practice. That left Elise and 17 others. How many people were in the class normally?
8. A jar contained 50 jellybeans. 10% of the jellybeans were strawberry flavoured. Liam only liked the black jellybeans, so he ate all 8 of them. How many jellybeans were **not** black or strawberry? What percentage is that?

19. Introducing ratios

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

Ratios are one of the most commonly used mathematical tools in the real world. They are used to represent the size of proportions in relation to each other, e.g. proportion of ingredients in a recipe, the length of fabric needed for making a curtain compared to the length of the window.

Explore

Use the worked example to identify how ratios are *similar* to and *different* from fractions.

- List the similarities and differences that you can find.
- How do you think ratios work?

What would change if you needed to cook 4 cups of rice?

- How much water would you need to use?
- How do you know? What did you do to work it out?

What would change if you needed to cook 1 cup of rice?

- How much water would you need to use?
- How do you know? What did you do to work it out?

Worked example

To cook most types of rice we use a ratio to determine how much water to add. For every **2** cups of rice, we add **3** cups of water. We use **5** cups of ingredients in total.

The **ratio** is 2:3

The **fraction** of rice is $\frac{2}{5}$

The **fraction** of water is $\frac{3}{5}$

Apply

Ratios focus on the parts **in relation to each other**. **Fractions** focus on the parts **in relation to the size of the whole**. Try using what you have worked out about ratios to determine how to express the following ratios.

1. When determining how much fabric to buy for curtains, we need to allow extra fabric for the folding or curving. We need 3 times the window length of fabric.
   a. Write this as a ratio.
   b. How much fabric would we need if the window was 4m long?
   c. If we bought 18m of curtain fabric, how long would the window be?
2. When cooking a meal, we need 1 cup of rice for every 2 adults.
   a. Write this as a ratio.
   b. How much rice would we need for 6 adults?
   c. If we cooked 2 cups of rice, how many adults could we feed?

Extend

3. In a class of students, 2 children played cricket for every 3 children who played soccer.
   a. Write this as a ratio.
   b. If there were 25 children in the class in total, how many played soccer and how many played cricket? How did you work it out?
4. A bag of lollies had a ratio of red: yellow: blue of 4:3:2
   a. If there were 9 yellow lollies, how many red and blue were there? Explain why.
   b. If there were 36 lollies altogether, how many would there be of each colour?

Sample

20. Ratios and recipes

MULTIPLICATIVE THINKING PROPORTIONAL REASONING

We only write ratios using whole numbers. We wouldn't write 1:1 ½ as a ratio of rice to water (1 cup of rice needs 1 ½ cups of water). Instead, we would double both numbers and write it as 2:3 (2 cups of rice need 3 cups of water). When writing ratios, we use the same measuring unit for each part (e.g. using cups: cups, not cups: litres).

Experiment

Examine the recipe for playdough provided. The ratio of flour, salt and oil is easy to calculate as all the ingredients are measured in cups. The oil is more difficult as it is measured in millilitres. You will need to know that 1 cup holds 250mL.

Work out a ratio for the ingredients using the same base measurement.

Adjust your ratio so that the oil is 1. How do your other ingredients change?

Recipe for playdough

- 2 cups plain flour
- 1 cup salt
- 25mL oil
- 1 cup cold water
- Food colouring if desired

Mix flour and salt.
Add wet ingredients.
Knead until combined.

Explore

Stimulus questions

- How many millilitres of water is used in the original recipe?
- How much more water is used than oil?
- How much more flour is used than oil?
- How much more salt is used than oil?
- If we used only 1mL of oil, how much would we use of the other ingredients?

What have you tried? What has worked or not worked?

Evaluate

Does your ratio use only whole numbers?
Does your ratio use the same system for measurement?
How reasonable is your answer?
Change anything that you need to.

Explain

Share your answers as a group. You may have used different methods to work out the answers. Share how you did it and try to find similarities and differences in your strategies.

Extend

A kindergarten wanted to make roughly 1 cup of playdough for each child. There are 18 children in the class. How much of each ingredient should they use?

Sample

21. Simplifying ratios

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

When we write ratios, we try to keep the numbers as small as possible. Rather than writing a ratio as 2:6, we would halve both numbers and write it as 1:3. This is called "simplifying". It requires thinking about whether the numbers in the ratio have any common factors. In the previous lesson, when we rewrote the recipe for playdough as a ratio where the oil was 1 part we were "simplifying the ratio".

Stimulus questions for simplifying ratios: 25 mL oil to 1 cup of water

- Do we need to change the measurement system to be the same?
  Yes: 25mL oil to 250mL water. This is 25:250
- Is there a common factor between the amounts in the ratio?
  Yes: both can be divided by 25.
- What simplified ratio could we use?

Set A: Practice and simple application

1. A two-stroke engine uses 25mL of oil for every 1L of petrol. Write the ratio of oil: water.
2. I am 1.62m tall. My hand print measures 18cm. Write the ratio of hand print: height.
3. A retailer pays $20 for product from a wholesaler. The retailer sells the product to customers for $28. Write the ratio of wholesale price: retail price.
4. I poured 50mL cordial into a one-cup glass, then topped it up with water. Write the ratio of cordial: water.
5. I poured 100mL cordial into a one-cup glass, then topped it up with water. Write the ratio.
6. I poured 40mL cordial into a one-cup glass, then topped it up with water. Write the ratio.

Complex application

A backyard swimming pool needs around 2 parts per million of chlorine to water.
This is a ratio of roughly 2: 1 000 000 or 1: 500 000

1. If a pool held around 25m^3 of water, how much concentrated chlorine would be needed? Use the information provided. Explain how you did it. Show your working.

Pool chlorine can be bought in liquid, granule or tablet form. If we were starting from a chlorine level of 0ppm, we would need to add 120g of pool chlorine granules per 10m^3 of water to achieve a concentration of about 2ppm. If we used liquid pool chlorine, we would use 100mL per 10m^3 to achieve the same concentration.

MULTIPLICATIVE THINKING

- 1 mL is the same as one cubic cm (picture a single/ unit MAB cube).
- 1 000 mL is the same as 1 000 cm^3, or 1L (picture a 1000 MAB cube, or 1 L of milk).
- 1 000 000 mL is the same as 1 cubic metre (or 1 kL).

2. My 25m^3 pool has a concentration of 1ppm chlorine. I want to get it to 2ppm. How much liquid or granule pool chlorine should I add to take the concentration to 2ppm? Explain how you did it. Show your working.

Sample

22. Dividing quantities into ratios

MULTIPLICATIVE THINKING PROPORTIONAL REASONING

Sometimes, we have to use the total number of items and a ratio to work out how many items there will be of each type. This is called dividing quantities into ratios.

Experiment

In our introductory lesson on ratios, the last question asked you to work out how many of each colour of lolly there would be when you knew how many lollies were in the bag and the ratio of colours. This is called dividing quantities into ratios. In this lesson we will learn a strategy for how to calculate the number of each item, using a **Relationship Table**.

The question you answered previously was:
A bag of lollies had a ratio of red: yellow: blue of 4:3:2. If there were 36 lollies altogether, how many would there be of each colour? The answer was 16 red, 12 yellow and 8 blue.

Red	Yellow	Blue	Total
4	3	2	9
			36

How did we change the 9 into a 36? Apply that same change to all other numbers to see what happens.

Explore

Stimulus questions

- If we add up the numbers in the original ratio, what is the total?
- How is that number related to 36?
- How could we have used that number to change our 4 red into 16 red?
- Does that same process work for the other numbers?

Try to summarise a process for calculating this number, then using this number to divide the quantity of 36 into the ratios.

Evaluate

Try using your method to solve these questions and evaluate how well it works.

- A bag had 30 lollies in the ratio of 2 red to 3 blue. How many were there of each?
- A bag had 24 lollies in the ratio of 2 red to 1 blue. How many were there of each?

How well did your method work?

Explain

Write a simple process for dividing a quantity into a ratio. Try to explain your process in as few steps as possible, making sure that it always works.

Extend

In the 2016 Australian census, 52% of respondents identified as Christian, 30% stated that they had no religion, 10% of people chose not to answer the question, and the remaining 8% identified mostly with Islam, Buddhism, Hinduism Sikhism and Judaism.

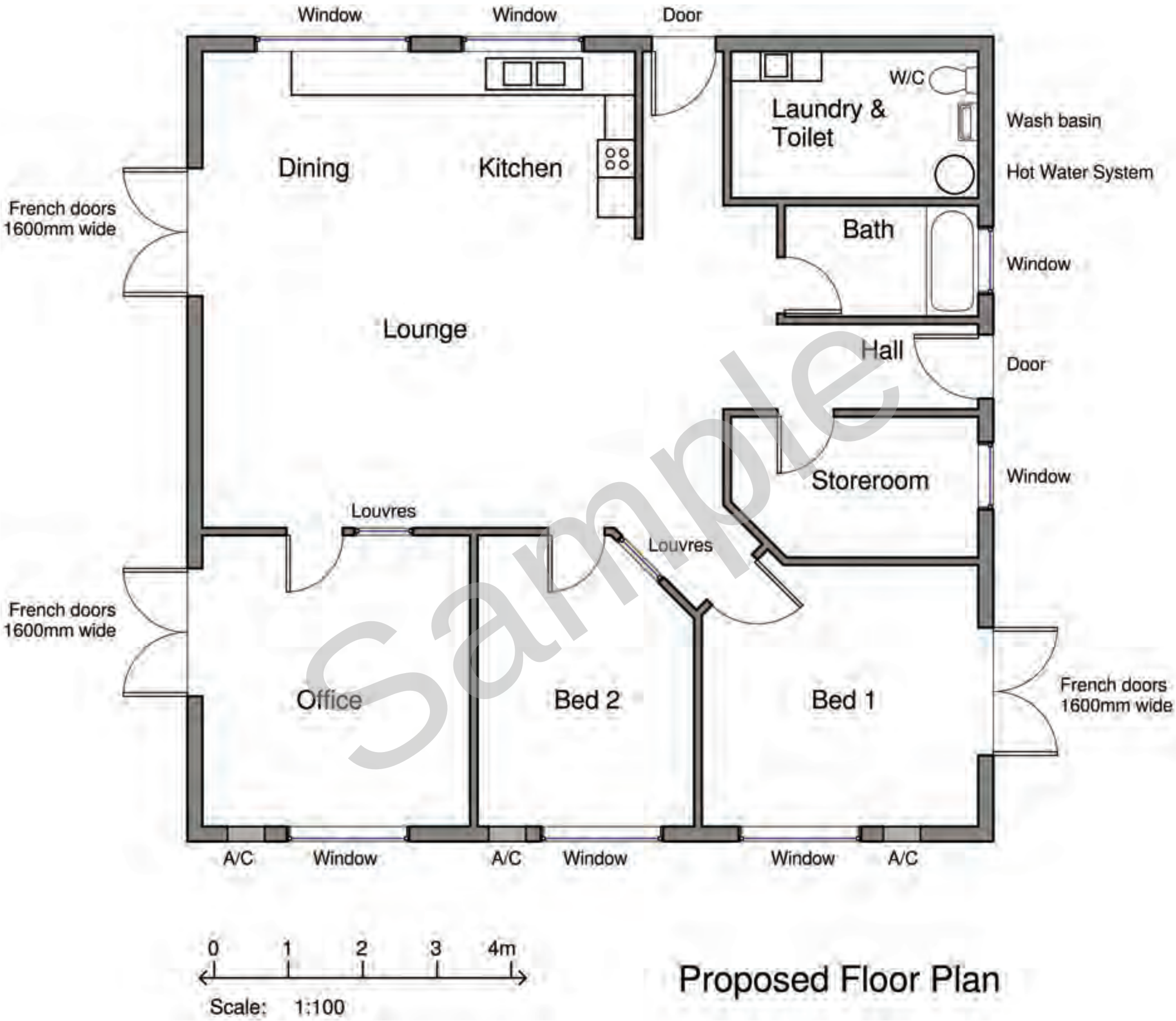
1. Write this as a ratio of Christians: No religion: No answer provided: Other religions, using everything you have learned about ratios so far.
2. Currently the Australian population is around 25 million. Use this to predict how many Australians there would be for each of the religions mentioned.

Sample

23. Ratios as scales

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

One of the most common applications for ratios is for scales on maps and plans. The diagram below shows a proposed floor plan for a house. The scale is given as a ratio at the bottom. In this case, the scale means that 1cm on the plan is the same as 100cm (or 1m) in real life.



Set A: Practice and application

1. Give the dimensions for the following rooms: office, bath, laundry.
2. Using the same scale, draw your desk. Commonly house plans are drawn using a scale of either 1:100 (above) or 1:50. Use a scale of 1:50 to draw your desk. Compare the drawings.
3. A city road map was drawn using the scale 1:50 000. Explain what this means. If you drove a road that was 4km long, how far would that be on your map?
4. Walking maps tend to use a scale of 1: 25 000. How many cm would be required to show a walking distance of 1km? Why is this important? Why would this scale be used?

24. Rates are used for different units of measure

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

Rates are very similar to ratios in that they express the relationship of parts. However, whereas ratios compare the same unit of measurement, rates compare different units of measurement. For example, a student typed 80 words in 2 minutes. We usually express rates as "per one" of a measuring unit. This means that typing speed would be described in "words per minute" rather than words per two minutes. 80 words typed in 2 minutes would be expressed as 40 words per minute (40wpm or 40 w/m).

Experiment

Examine the rates below. Work out how to express them properly. Use the commonly used rates information to help you work them out.

1. A car travelled 120km in 2 hours. Write the car's speed in km/h.
2. A car travelled 25km in 30 minutes. Write the car's speed in km/h.
3. A plumber earned $15 in 15 minutes. Write the plumber's pay rate per hour.
4. A lamp used 6kw of power in 3 hours. Write the lamp's energy consumption rate in kw/h.
5. A mobile phone user was charged 30c (plus a standard connection fee) for a 15 minute call to the USA. Write the phone charge as a rate per minute.

Commonly used rates

- Car speed: km per hour (kph or km/h)
- Pay rates, penalty rates and charge-out rates: $ per hour
- Currency exchange rates: amount per $1AUD
- Motor speed: revolutions per minute (RPM)
- Interest rates: interest per $1
- Energy consumption rates: kilowatts per hour (kw/h)
- Phone charges: $ per minute

Sample

What did you do to write each of the situations above as a rate "per one"? Try to explain the process in a way that can be easily simplified.

Explore and Evaluate

The **Relationship Tables** below might help you to work out a method for expressing unit rates correctly. Work through them until you think you have a method that works consistently. The questions below use the same numbers as above so that it makes more sense to you.

km	h
120	2
60	1

km	h
25 km	½ hour
	1

$	h
15	¼ hour
	1

kw	h
6	3
	1

Explain

How did you use multiplication or division in your Relationship Tables to write the rates properly? What processes did you use? Why did you have to start with the "per one" unit? Explain your system as simply as possible.

25. Calculating with rates: speed, rainfall, pay rates

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

Once we know a unit rate, we can use it to easily calculate. In this task, you will first calculate a unit rate using your method from last lesson, then apply it to real world situations.

Explore and Apply

Use **Relationship Tables** to answer the following questions.

1. An apprentice carpenter earned $80 in 5 hours. What was her pay rate? What would she have earned in a 38 hour week?

$	h
80	5
	1

So

$	h
	1
	38

2. A casual teacher earned $456 for a 6 hour day. What was her pay rate? What would she have earned in a week of work (30 hours)?

$	h
456	6
	1

So

$	h
	1

Sample

3. In 2019, Townsville received a record amount of rainfall over the ten days leading up to 6th February. According to the Bureau of Meteorology, 1257mm fell in 10 days[1]. What was the average rate of rainfall per hour for the whole ten days (240 hours)? Round the rate off to the nearest whole millimetre.

mm	h
1257	240
	1

4. In 1997, Andy Green set a new land-speed record when driving a twin turbofan jet-powered car. He averaged 1227.985 km/h over the course[2]. The course was 1 mile, which is 1.61km long. How long would it have taken him to drive the 1.61km? Try to answer in hours (this will be very small), then convert it into minutes and seconds using similar thinking. You should end up with a final answer of less than 5 seconds.

km	h
1228	1
1.6	

Or...

km	min
1228	60
1.6	

Or...

km	sec
1228	3600
1.6	

[1] http://www.bom.gov.au/climate/current/statements/scs69.pdf
[2] https://www.fia.com/

26. Converting between rates 1

MULTIPLICATIVE THINKING **PROPORTIONAL REASONING**

*In the real world, we often have to convert between different rates to make comparisons. Sometimes, we only have to convert one of the parts, so we perform one operation. This activity looks at converting one part only. Use **Relationship Tables** to help you.*

Experiment: Converting with one part

Speed in the USA is expressed in miles per hour (mph or mi/h) whereas, in Australia, we use kilometres per hour (kph or km/h). In this case, we need to convert the distance, but not the time as both are expressed "per hour". Look at the table on the left below and use it to work out how to change 60 miles per hour into km per hour in the table on the right.

distance	time	
1 mile	1h	unit rate
= 1.6 km	1h	

distance	time
60 miles	1h
km	1h

Explain

Complete these sentences:

1. To convert from 1 mile to km, we have to multiply by 1.6, so to convert from 60mph to kph we have to…
2. To convert from 1km to miles we have to… so to convert from kph to mph we have to…

Apply

Apply your process to convert the following rates. All of the questions require you to only convert the distance. To help you out here are some imperial to metric conversions. Each have been rounded to a single decimal place.

1 mile = 1.6km   1 yard = 0.9m   1 inch = 2.5cm

Animal	Top speed in imperial units[3]	What do I do?	Speed in metric units
Sea star	0.6 miles per hour		km/h
Snail	30 inches per min		cm/min
Sloth	41 yards per day		m/day
Manatee	5 miles per hour		km/h
Slug	1.9 miles per hour		km/h

3. What is the same about your process each time? What would be different if we were converting from metric units to imperial units?
4. Convert 25cm/min to inches/min.
5. Convert 12m/day to yards/day.

[3] https://frontier.ac.uk/blog/2016/12/18/10-slowest-animals

Sample

27. Converting between rates 2

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

In the previous task, we converted one of the parts in a rate to a different unit (e.g. miles per hour to kilometres per hour). In this task, we will convert both of the units (e.g. km/h to m/sec). That requires performing two operations rather than one. Use ***Relationship Tables*** *to help you.*

Experiment

The following tables show what happens when we convert 1km/h into m/sec. Use the tables to work out what operations were performed to change the rate.

Step 1: Distance

distance	time
1 km	1h
= 1000 m	in one hour

old rate

new rate

Step 2: Time

distance	time
1000 m	in 3600 sec
	in 1 second

Explore

1. What operation did we perform to turn 1km into 1000m? What did we multiply by?
2. What operation did we perform to turn 3600 sec into 1 sec? What did we divide by?
3. Check your answer. You should have found that 1km/h is the same as $0.2\underline{7}$ m/sec.

Evaluate

Check that your system is working by converting 60km/h into m/sec. You should get $16.\underline{6}$m/sec. If you are confident that it's working, try converting the following speeds from km/h into m/sec. Questions 6 and 7 convert from m/sec into km/h. Draw tables, if it will help, or just use operations if you find that easier. Round off your answers to a single decimal point.

4. 50km/h to m/sec
5. 80km/h to m/sec
6. 20m/sec to km/h
7. 10m/sec to km/h

Apply and Extend

The following questions are more complex as they require multiple steps. Remember to check your answers as you go.

8. I walked 12km in 2 hours. What is my speed in km/h? What is my speed in metres per minute? How far will I have walked after only 15 minutes?
9. I run 100m in 15 seconds. What is my speed in m/sec? What is my speed in km/hour?

Sample

28. Using rates to make comparisons

MULTIPLICATIVE THINKING PROPORTIONAL REASONING

*As rates are usually expressed "per unit", they are a great way of making comparisons. In this activity, you will apply what you have learned about rates to compare them. Use **Relationship Tables** to help you.*

Apply

Work out the rate for the price/kg for each of the following items and order them from the cheapest to the most expensive. Show your working.

1. A shopping centre sold salami in the following sizes:
   A. 200g knob of salami for $6.20
   B. 80g packet of pre-sliced salami for $3.25
   C. Pre-sliced salami from the deli for $32 per 100g
2. Oranges were sold in the following units:
   A. A 3kg bag of oranges for $5.50
   B. Unpacked oranges for 60c each (oranges weigh on average 180g)
3. Tinned tomatoes came in the following sizes.
   A. 600g diced cherry tomatoes for $3.15
   B. 400g diced cherry tomatoes for $2.10

Extend

When discounts are applied to items, it changes their price rate. How would your answers to the questions above change if a 10% discount was applied to the most expensive item in each group? Show your working.

Sample

29. Review "percentage of" an amount

PROPORTIONAL REASONING

In the previous chapter, you learned to work with percentages. In this activity, you will learn three strategies to calculate percentages so that you can choose which makes the most sense to you.

Strategy one: Of A Dollar

Percent and cent both have to do with 100. Percent means "out of 100". 100 percent is equal to one whole, just like 100c is equal to one dollar. That means that we can think of percentage as cents.

Worked example: Find 40% of 2

- 40% means 40c out of $1.
- In this case we have 2 rather than 1.
  So, I need 2 lots of 40c, or 80c. That's 0.80.
- So 40% of 2 = 0.8

PROPORTIONAL REASONING
RELATIVE SIZE

Consider one quarter *of a dollar*:

- How many cents in one quarter of a dollar?
- The number of **cents** is the same as one quarter as a per**cent**. Cent means hundred.
- Write one quarter of a dollar as dollars and cents with a decimal point. That is one quarter as a decimal number.
  $\mathbf{\frac{1}{4} = 25c = 25\% = 0.25}$
- Likewise, the percentage is the number of cents. So 15% is the same as 15c or 0.15.

Strategy two: what of means

What operation does "of" usually mean? For example, 2 packets of 10 lollies. Once we know this, it is simply a matter of using that operation on the numbers in the question.

Worked example: Find 40% of 2

- How do we write 40% as a decimal number? 0.4
- What operation does "of" mean?
- Use that operation with your calculator to calculate 40% of 2. The answer should be 0.8

Strategy three: make the percent into an easier fraction

For key percentages it is usually easiest to think of a fraction. For example, 50% of means half.
Key percentages to know include: $50\%=\frac{1}{2}$ $25\%=\frac{1}{4}$ $75\%=\frac{3}{4}$ $10\%=\frac{1}{10}$ $20\%=\frac{1}{5}$ $5\%=\frac{1}{20}$

Worked example: Find 40% of 2

- 40% is 4 lots of 10%. So, if I find 10% of the total, I can just multiply that by 4.
- One tenth of 2 is 0.2, and 4 lots of that is 0.8

Evaluate

Use all three strategies to answer each of the questions below. The answers are provided. Which strategy makes the most sense to you for each question?

1. Find 30% of 5. The answer is 1.5
2. Find 75% of 8. The answer is 6
3. Find 50% of 50. The answer is 25
4. Find 20% of 60. The answer is 12

Sample

Set 1

1. Order these lengths from smallest to largest: 1.45m, 212cm, 1.450m, 0.45km
2. Use leading-digit approximation to estimate the answer to 273 + 685, then calculate the answer.
3. Write 0.75 as a fraction and a percentage.
4. 14 + 17 – 27 ÷ 9 = _____
5. I pay $83.56 each week for my car loan. Approximately how much will I pay over the period of a year?
6. You earn the national minimum wage of $38 521 Gross. After paying tax of $4066, how much do you earn Net? How much is this each week?
7. After paying your car loan, petrol costs of $20/week, and basic registration/insurance costs of $25/week, how much do you have left?
8. If you live in a capital city and work in the city, parking is very expensive. A 2013 report into commuter costs and potential savings[40] showed that the cost of owning a car and commuting 5 days per week was, on average, $11 031 per year. By comparison, the annual average cost of commuting by public transport was only $1 607. In this scenario, how much would you save each year by choosing not to buy a car?
9. Using the figures above, what percentage of your Net pay would be spent on commuting if you owned and drove a car? What percentage would be spent if you used public transport instead?

Sample

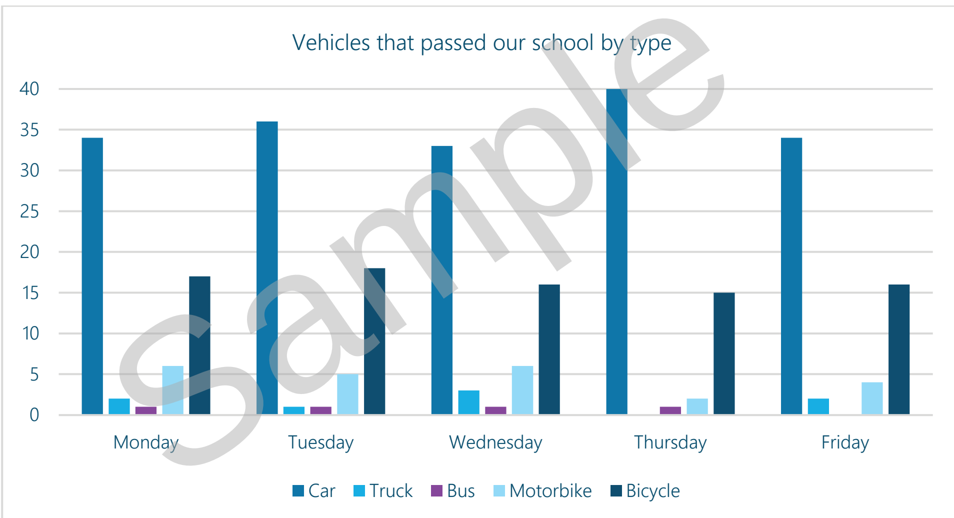
Set 2

1. Add 9.2 metres to 12 150 millimetres.
2. Use leading-digit approximation to estimate the answer to 746 – 428, then calculate the answer.
3. What fraction of $1 is 45 cents?
4. 11 + 3^2 – 3 x 5 = ____
5. The population[41] for Queensland in December, 2018 was 5 052 800, for New South Wales 8 046 100 and for Victoria was 6 526 400. Use leading-digit approximation to estimate how many people live in these three states altogether.
6. How many more people live in NSW than in QLD?
7. If the Australian population is 25.18 million (December, 2018), *roughly* what fraction or what percentage lived in Queensland?
8. According to the ABS, Australia's population grows by about 1.6% per year. If the population was 25.18 million in 2018, what will it be this year?
9. The time is 2:45pm. I start work at 5:20pm. How long until I start work?

[40] https://ara.net.au/sites/default/files/Commuter-costs-potential-savings-report-FINAL%20%281%29.pdf
[41] https://www.abs.gov.au/AUSSTATS/abs@.nsf/allprimarymainfeatures/1988DE98D5424933CA258479001A75A5?opendocument

Set5

1. Write these capacities in order from smallest to largest: 12.605kL, 12 065L, 12 560 741mL, 12.506kL
2. 3460 + 5137 – 125 = ______
3. The local dog park needs a new fence. Each panel is 1.25 meters long. The length of the rectangular park is 15 panels long and the width is 8 panels long. What is the perimeter of the park?
4. What fraction of a metre is 70 centimetres?
5. 4 (623 – 147) = ______
6. Is the answer reasonable? Use approximation to check. 487 x 32 = 15 584
7. A rectangle has an area of $36m^2$. Give 2 options for its dimensions.
8. The following graph shows vehicles driving past our school each day for one week. How many motorbikes passed by altogether? What is the average (mean) number of motorbikes to pass by each day?

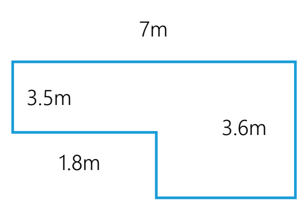


Sample

9. How many vehicles passed by our school altogether on Friday?

Set 6

1. 640 ÷5 = ______
2. This is a drawing of the coop I am building for my chickens. What length of chicken wire will I need to buy to enclose it?



3. What is 25% of $80? What fraction of $80 does it represent? Write it as a decimal.
4. True or False? Explain your thinking
   11 (12 + 37) = 11 x 12 + 407
5. Is the answer reasonable? Use approximation to check. 161.5 ÷ 3.8 = 42.5