Why kids don't get division and how to fix it

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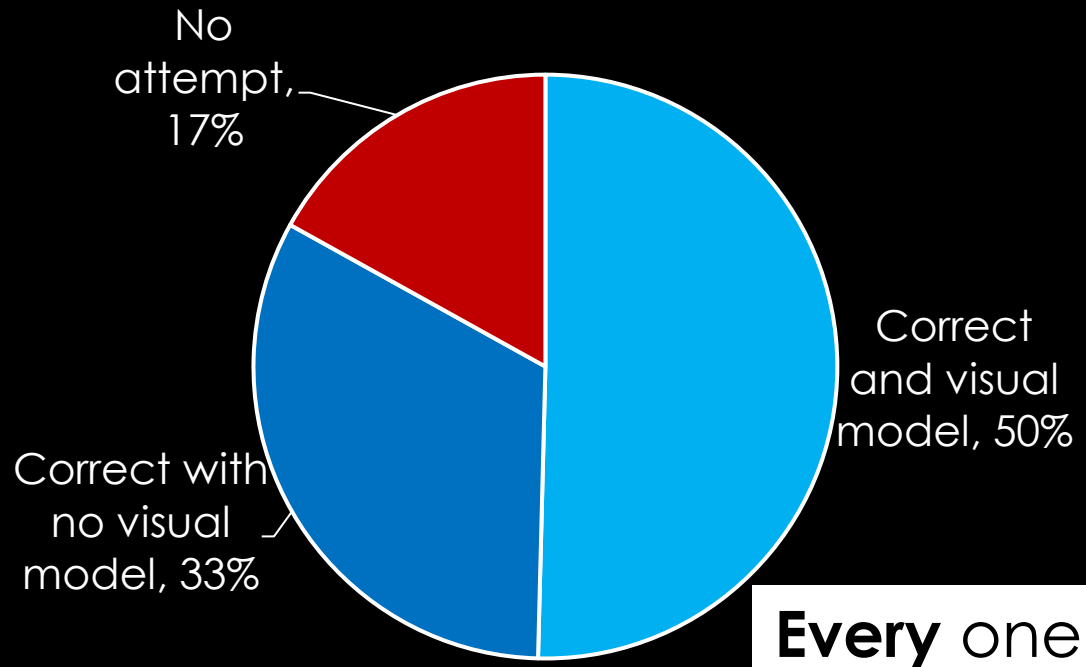
Today:

- Is division actually harder?
- Let's try a question
- Linking concepts and vocabulary
- Arrays vs area and the reason it matters
- Structural thinking
- Developmental sequence and student samples
- Alternative strategies

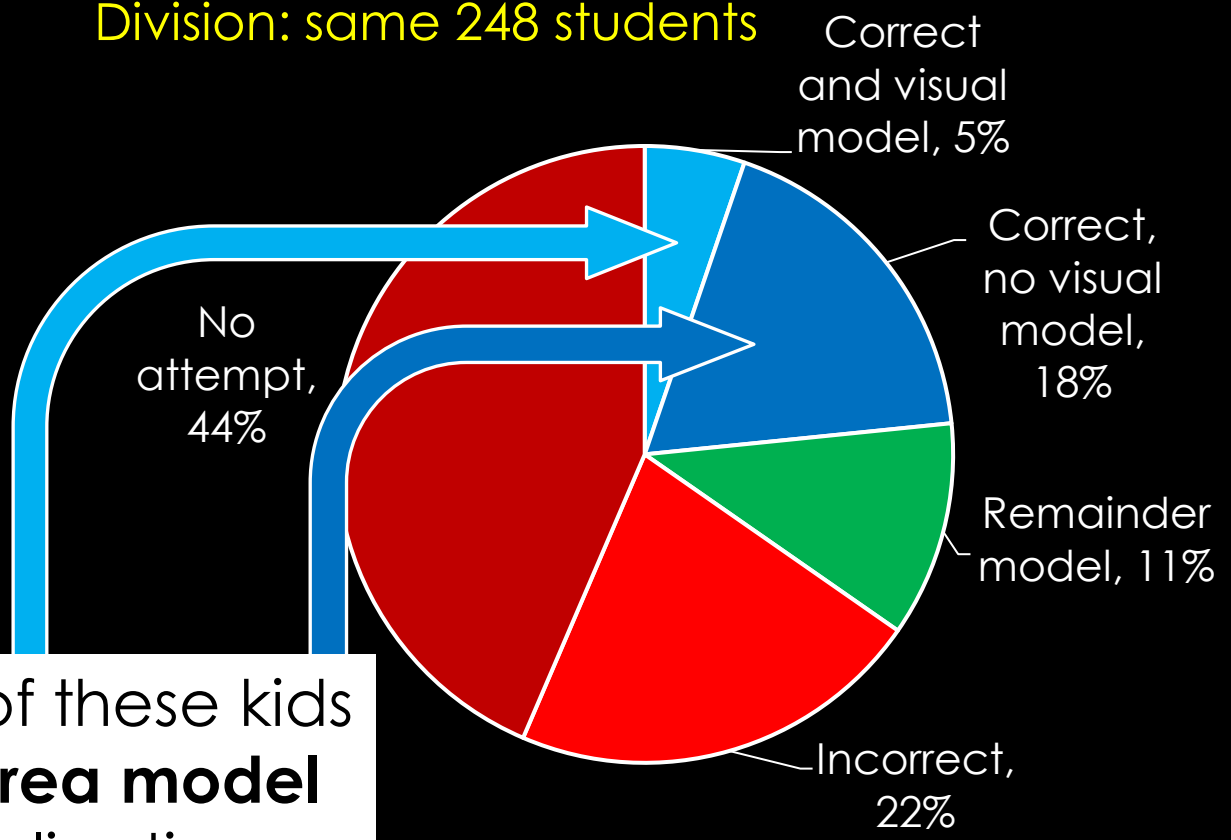
Is division harder than multiplication?

248 samples, Years 7-10, August 2020

Multiplication: 248 students



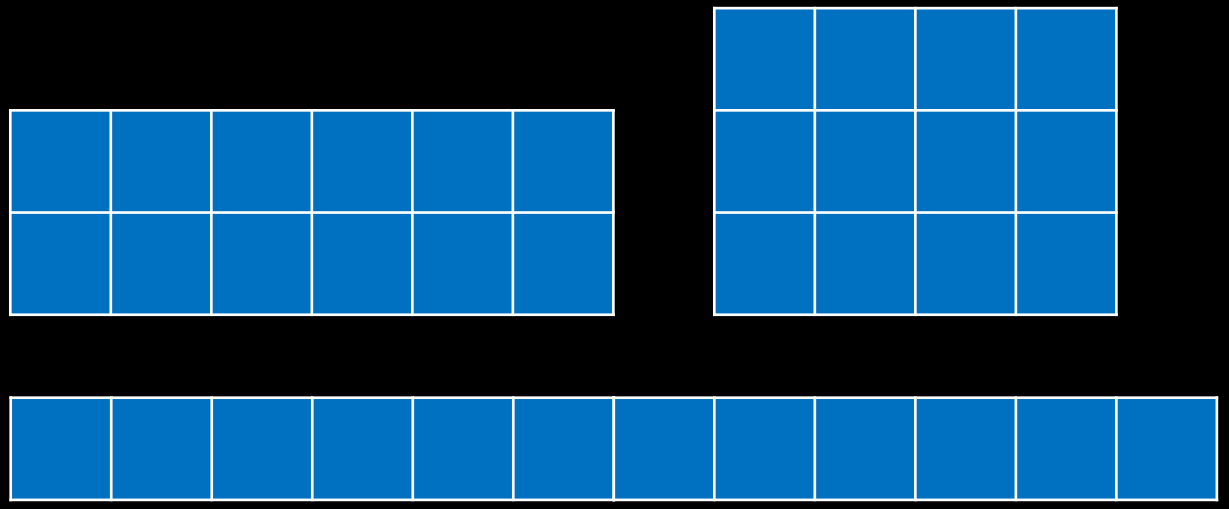
Division: same 248 students



Every one of these kids drew an **area model** for multiplication

Let's try it

Try drawing rectangles and cutting them to make 12 squares (actual squares)

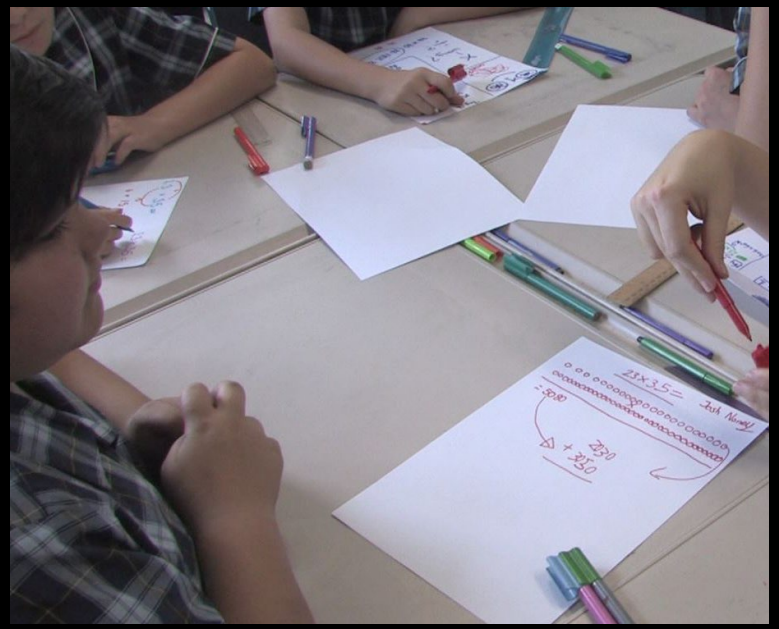


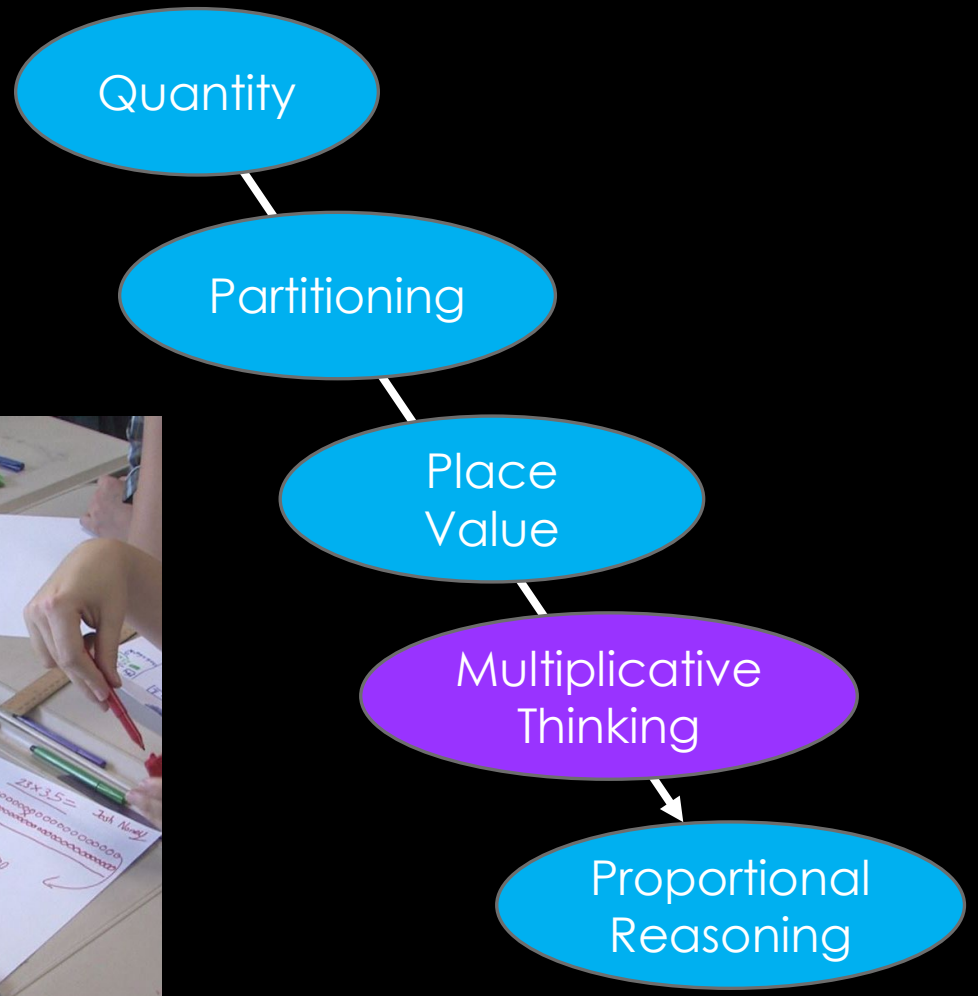
Was it harder than you expected to get the proportions right so that you made squares?

Key Number concepts

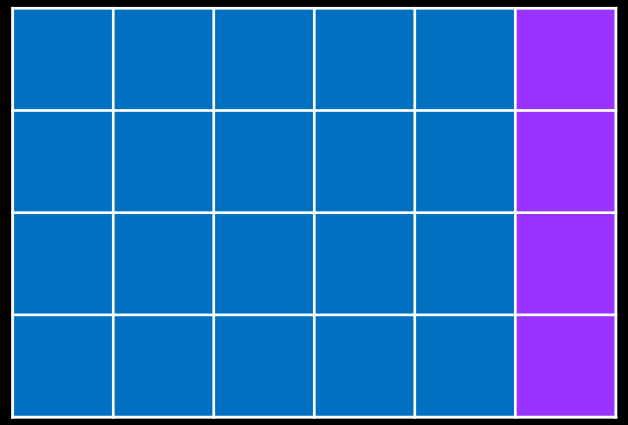
Multiplicative thinking:

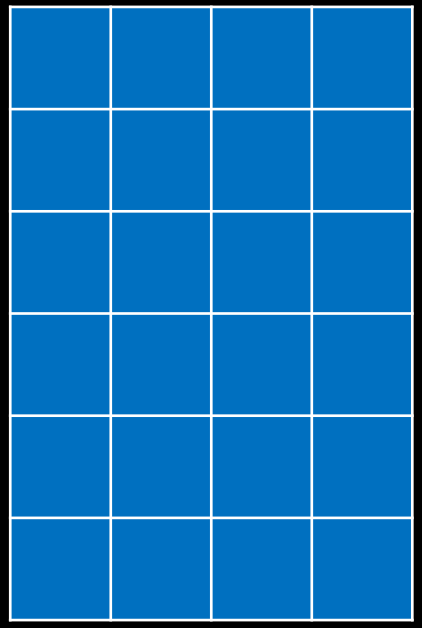
Thinking in multiples and **structuring** those multiples into areas, regions or arrays rather than groups.





Links to concepts and vocabulary





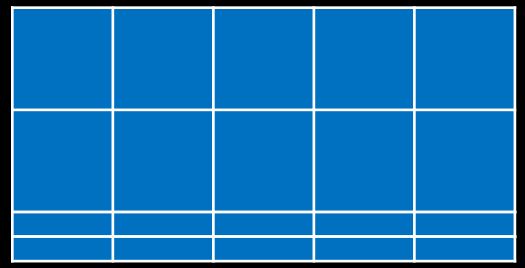
- Where are the factors? Where is the multiple?
- How could you tell if a number was composite?
  How could you tell if it was prime?
  How could you tell if it was square?
- Arrays help students to understand the **commutative property** of multiplication.
- Arrays help students to understand the **distributive property** of multiplication.
- Arrays link with area and volume.
- Arrays as division link with fractions and stats.
- Area links to algebra.

Why division links with area, not arrays

Find your arrays of 12

- Find the 3x4 array
- Write the factors on the side and the multiple in the middle
- Rub out the bottom and right-hand lines
- Now let's try drawing 12 ÷ 5

$$\begin{array}{r|l} & 4 \\ \hline 3 & 12 \end{array}$$

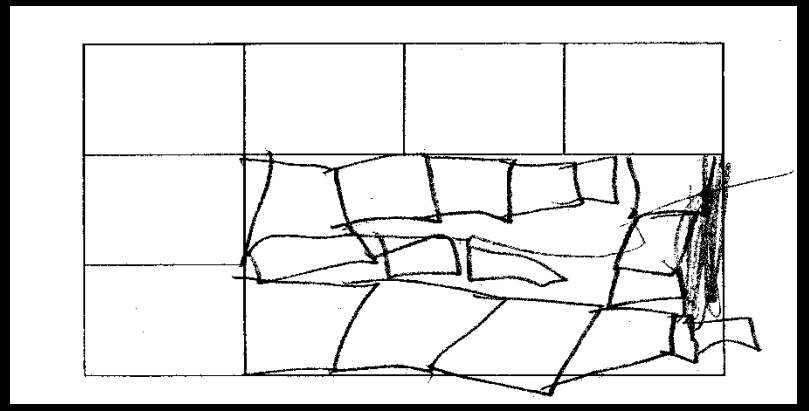


Structural thinking
Joanne Mulligan: PASA

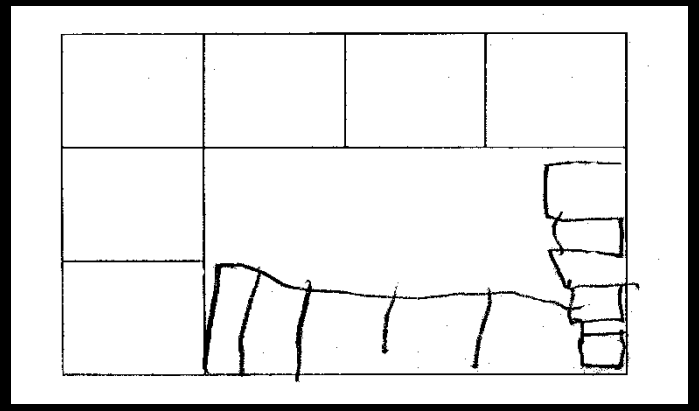
Look it up:

Mulligan, J. T., (2010). Reconceptualising early mathematics learning. ACER Press

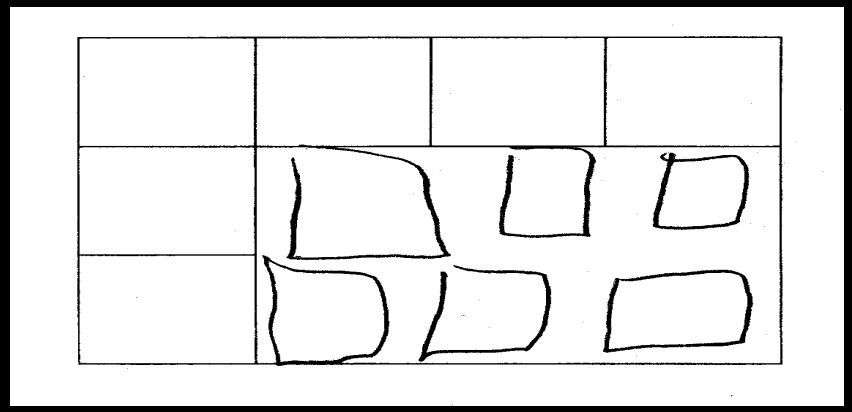
1. Prestructural
2. Emergent
3. Partial structural
4. Structural



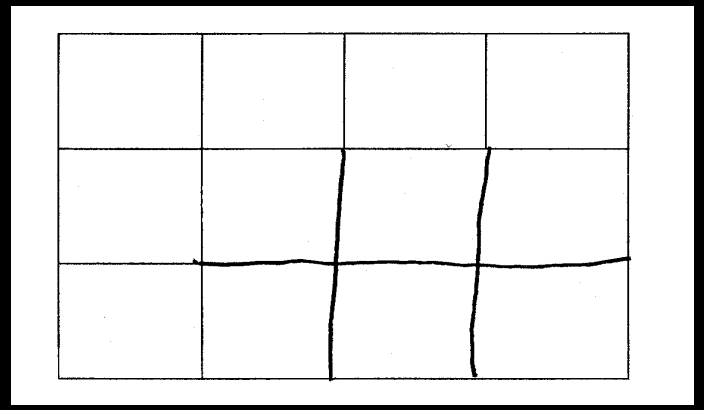
Prestructural



Emergent



Partial

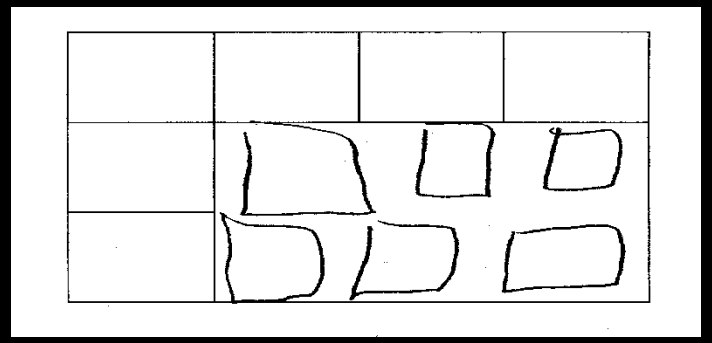


Structural

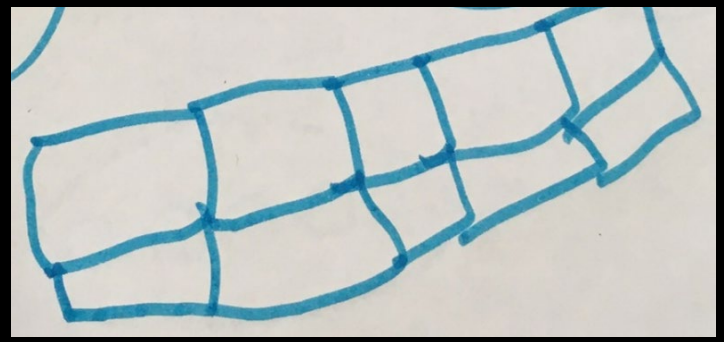
Developmental sequence from Tierney

Numbers to: 4, 6, 12, teens, 2 digit, 3 digit

Quantity to 6, Partition 4 → Quantity to 12, Partition to 6 small **arrays** (to 6) → Quantity to 20, Partition to 12 small **arrays** (to 12) → Partition to 20, Quantity to 100, small arrays but **connected** (to 20) → Quantity and Partition to 100, **structural** or **area** model to 20 → Relative size to 10 → **Area** to 5 x 10, Base-two fractions, 3-digit place value, Rel size to 100, division with remainders → **Area** to 10 x 10, Rel Size to 1000 Simple fractions, division with fractions and decimals → Fractions >1, decimals conversions Number properties AS algebra → Percentage, balance and algebraic generalising



Partial Array



Connected squares

Structural Area

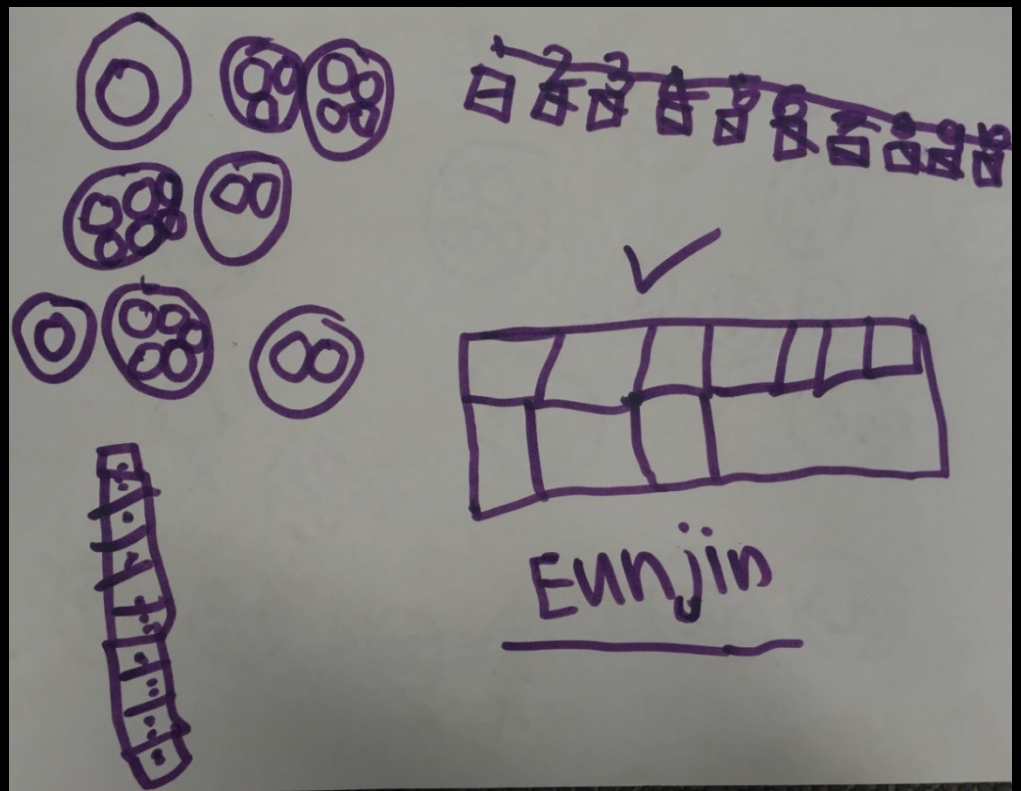
Quantity to 6, Partition 4

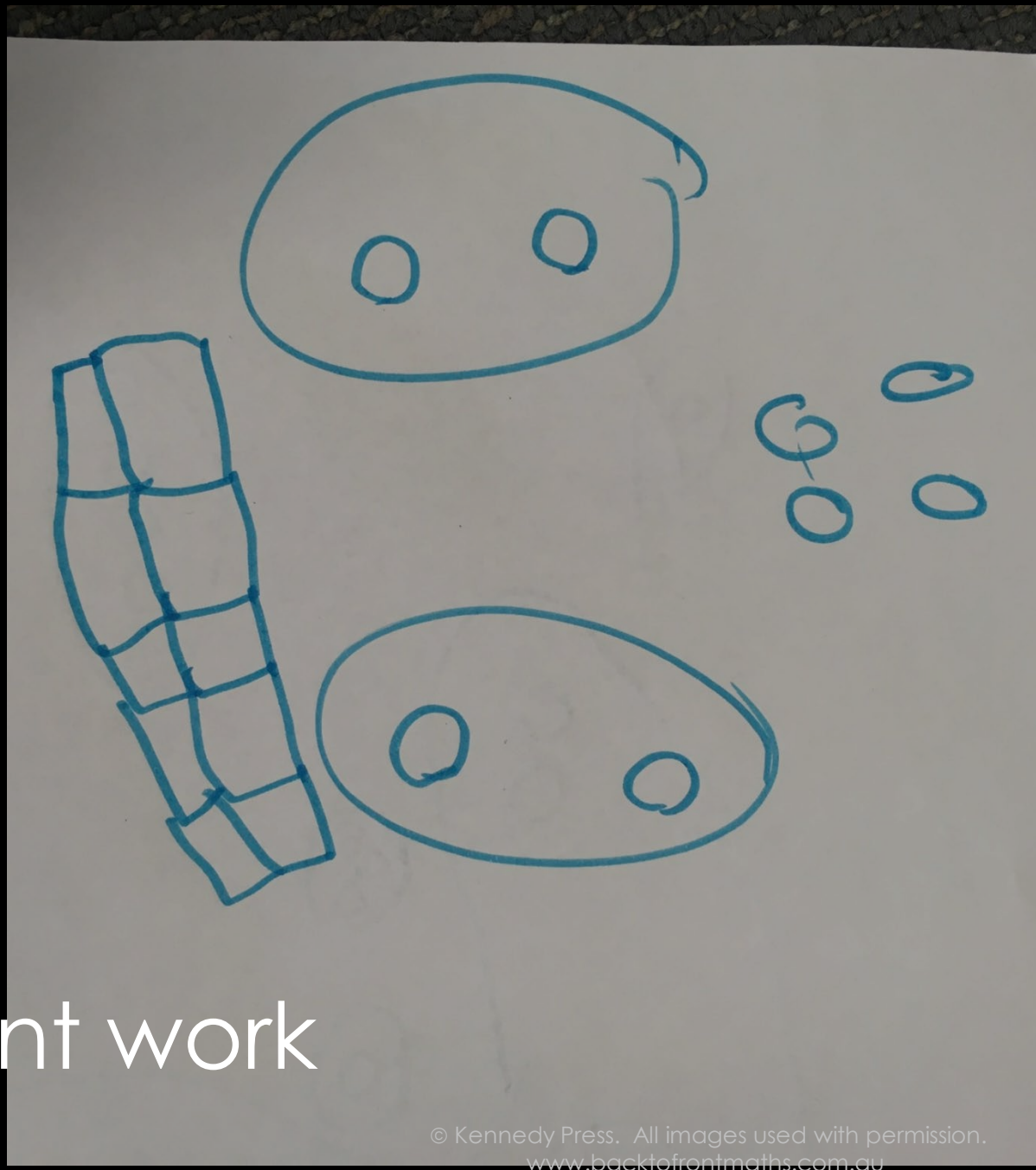
Quantity to 12, Partition to 6 small **arrays** (to 6)

Quantity to 20, Partition to 12 small **arrays** (to 12)

Partition to 20, Quantity to 100, small arrays but **connected** (to 20)

Quantity and Partition to 100, **structural** or **area** model to 20
Relative size to 10





Student work

Quantity to 6, Partition 4

Quantity to 12, Partition to 6 small **arrays** (to 6)

Quantity to 20, Partition to 12 small **arrays** (to 12)

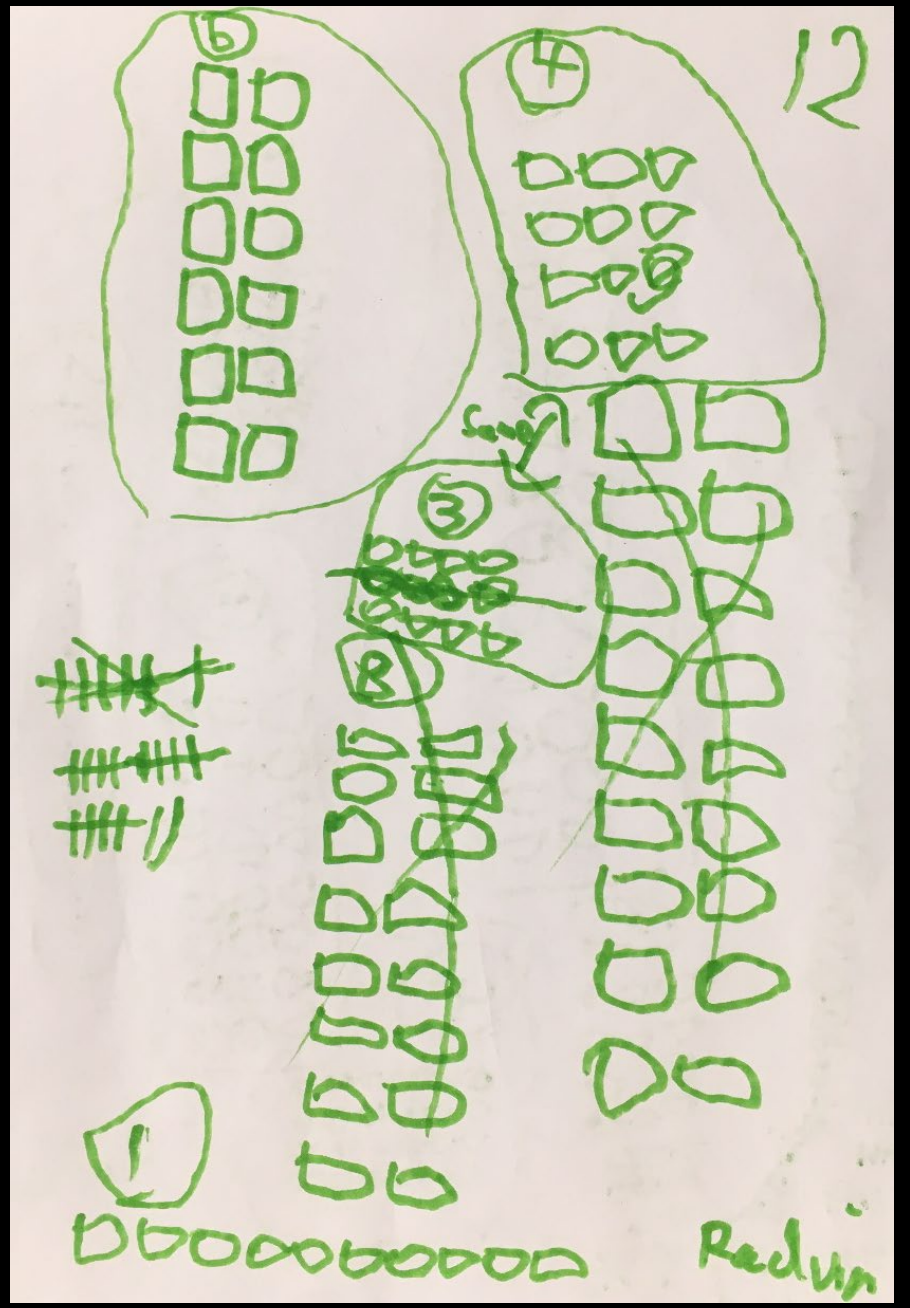
Partition to 20, Quantity to 100, small arrays but **connected** (to 20)

Quantity and Partition to 100, **structural** or **area** model to 20

Relative size to 10

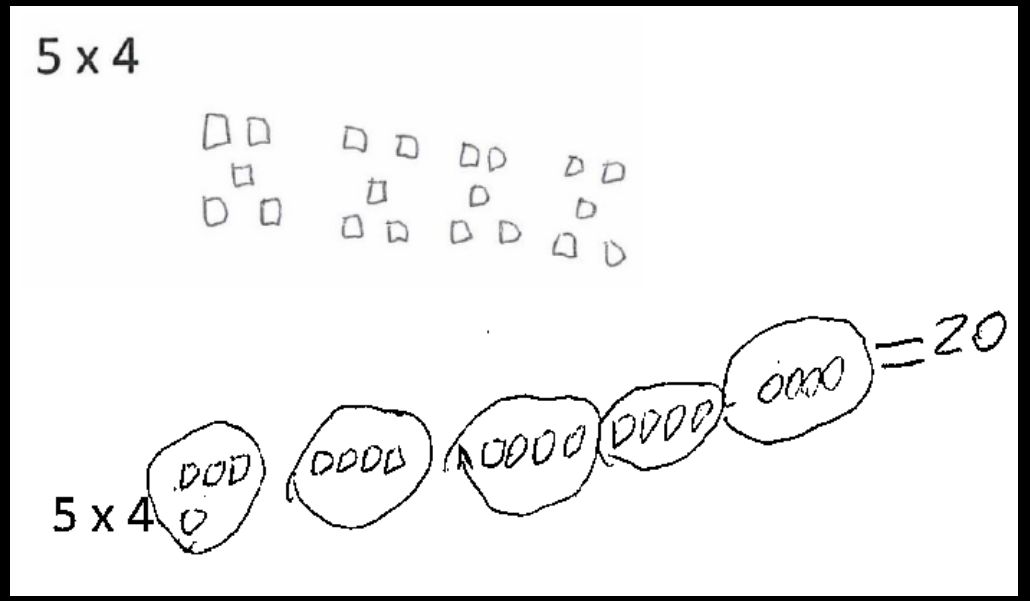
Area to 5 x 10, Base-two fractions, 3-digit place value, Rel size to 100, division with remainders

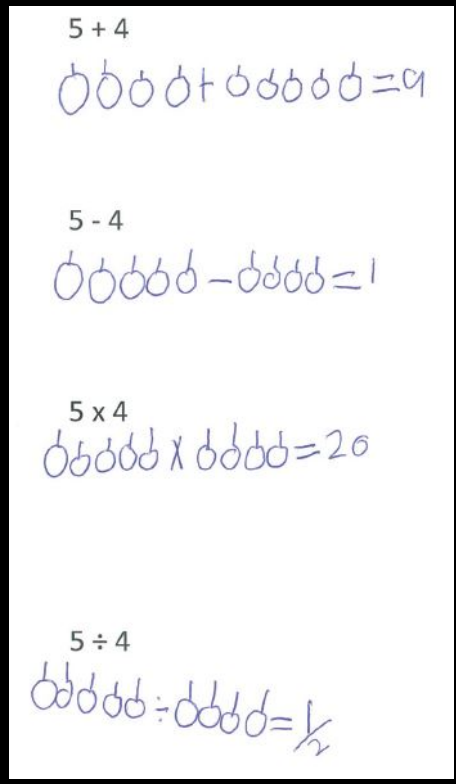
Student work 2



Quantity to 6, Partition 4 → Quantity to 12, Partition to 6 small **arrays** (to 6) → Quantity to 20, Partition to 12 small **arrays** (to 12) → Partition to 20, Quantity to 100, small arrays but **connected** (to 20) → Quantity and Partition to 100, **structural** or **area** model to 20 Relative size to 10 → **Area** to 5 x 10, Base-two fractions, 3-digit place value, Rel size to 100, division with remainders → **Area** to 10 x 10, Rel Size to 1000 Simple fractions, division with fractions and decimals → Fractions >1, decimals conversions Number properties AS algebra → Percentage, balance and algebraic generalising

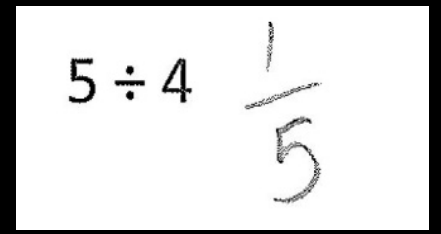
Student work 3

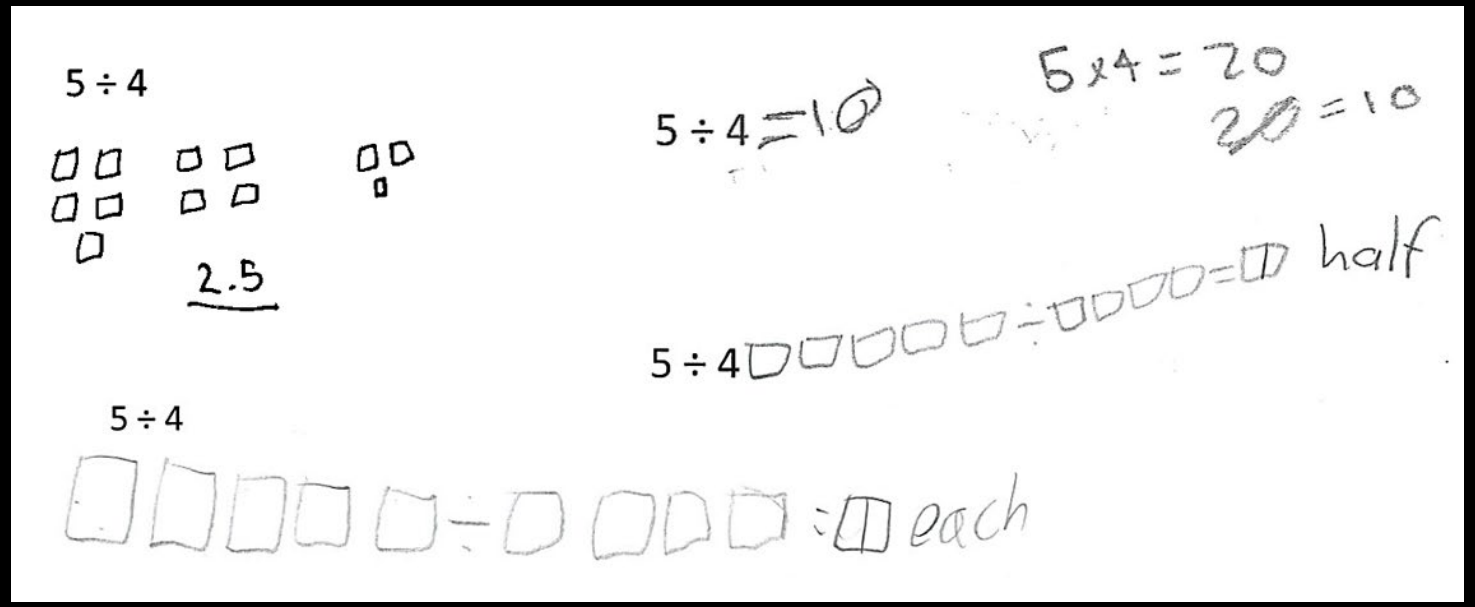


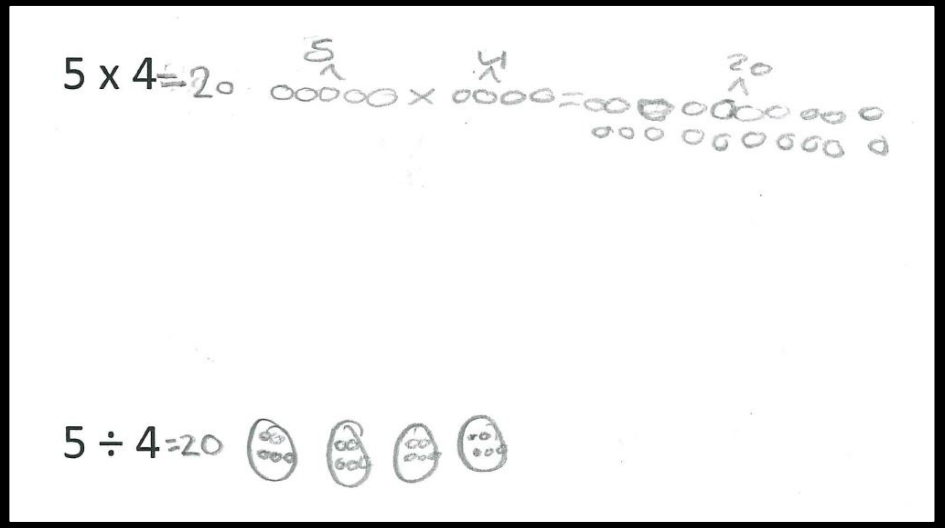


Quantity to 6, Partition 4 — Quantity to 12, Partition to 6 **small arrays** (to 6) — Quantity to 20, Partition to 12 **small arrays** (to 12) — Partition to 20, Quantity to 100, **small arrays but connected** (to 20) — Quantity and Partition to 100, **structural** or **area** model to 20 Relative size to 10 — **Area** to 5 x 10, Base-two fractions, 3-digit place value, Rel size to 100, division with remainders — **Area** to 10 x 10, Rel Size to 1000 Simple fractions, division with fractions and decimals — Fractions >1, decimals conversions Number properties AS algebra — Percentage, balance and algebraic generalising

Student work 4







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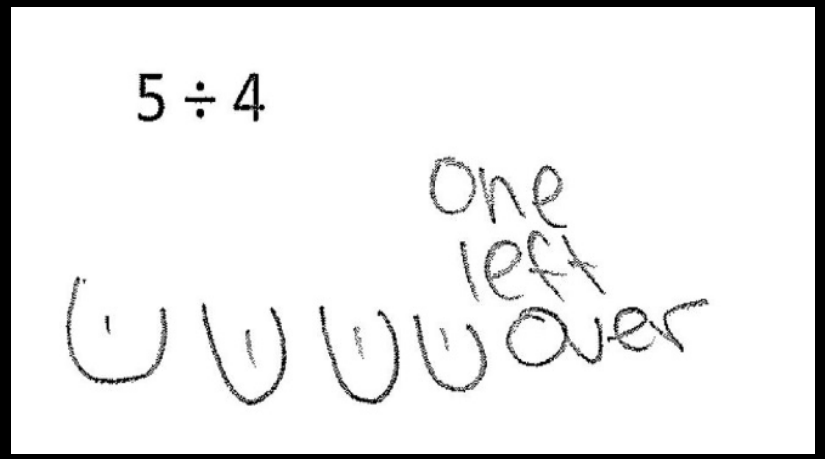
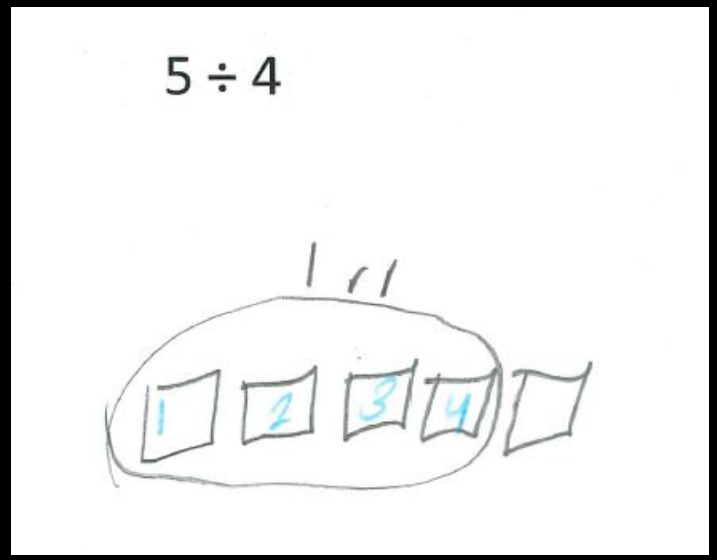
Student work 5





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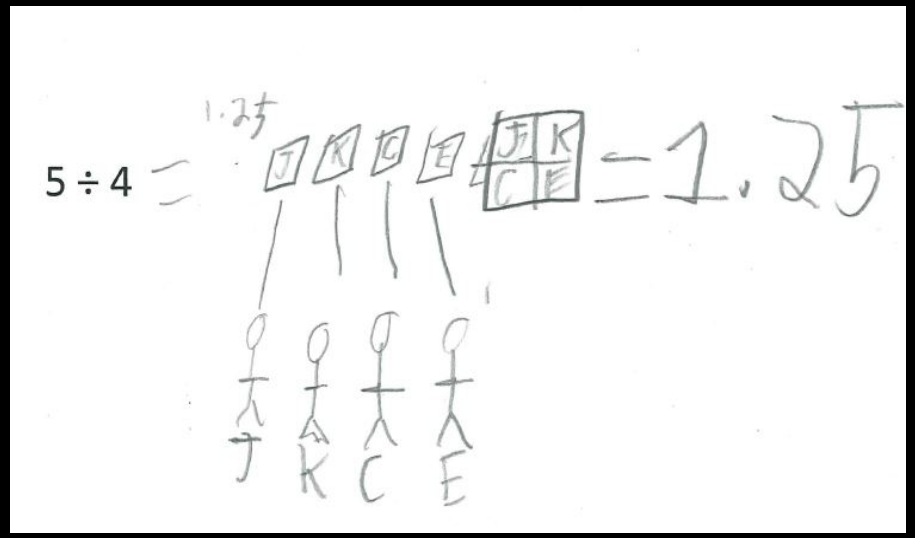
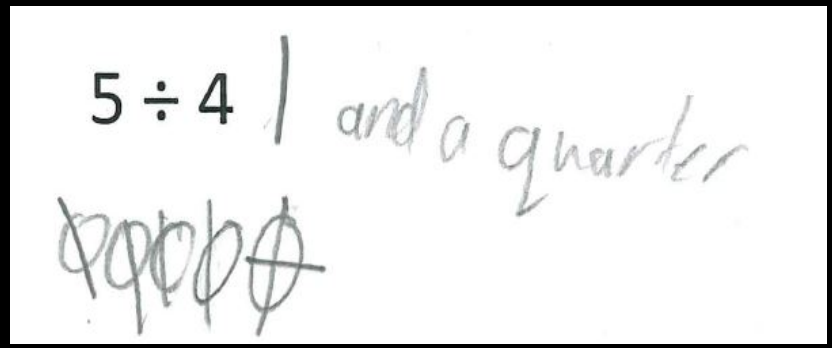
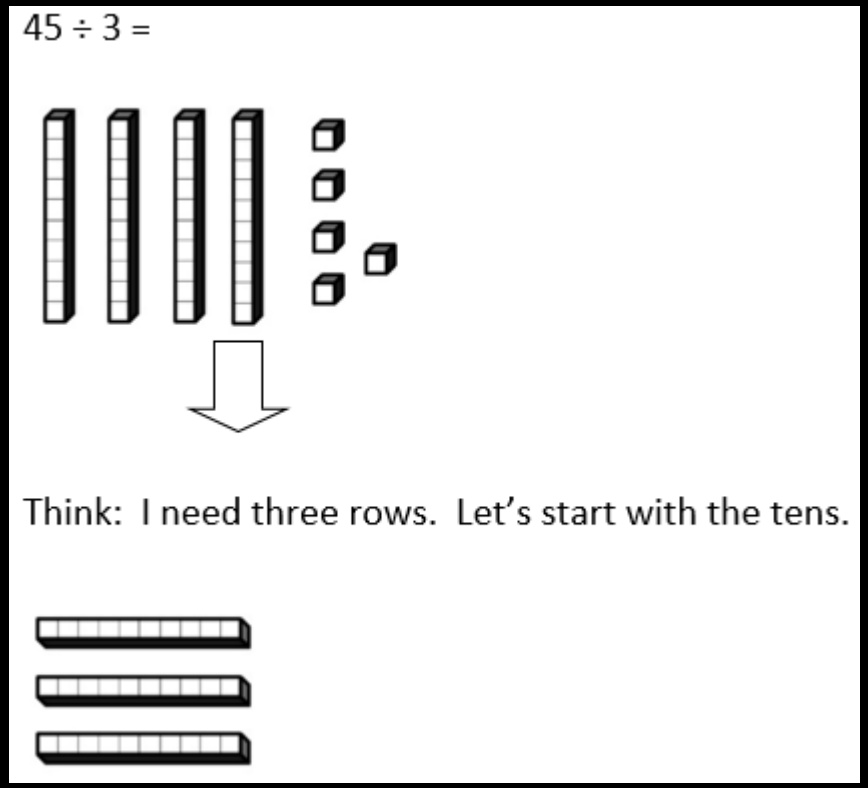
Student work 6





Dividing larger amounts

45 ÷ 3
Think: what would this look like as an array?
How can I put 4 tens into 3 rows?
What should I do with the left over blocks? How many will be in each row?



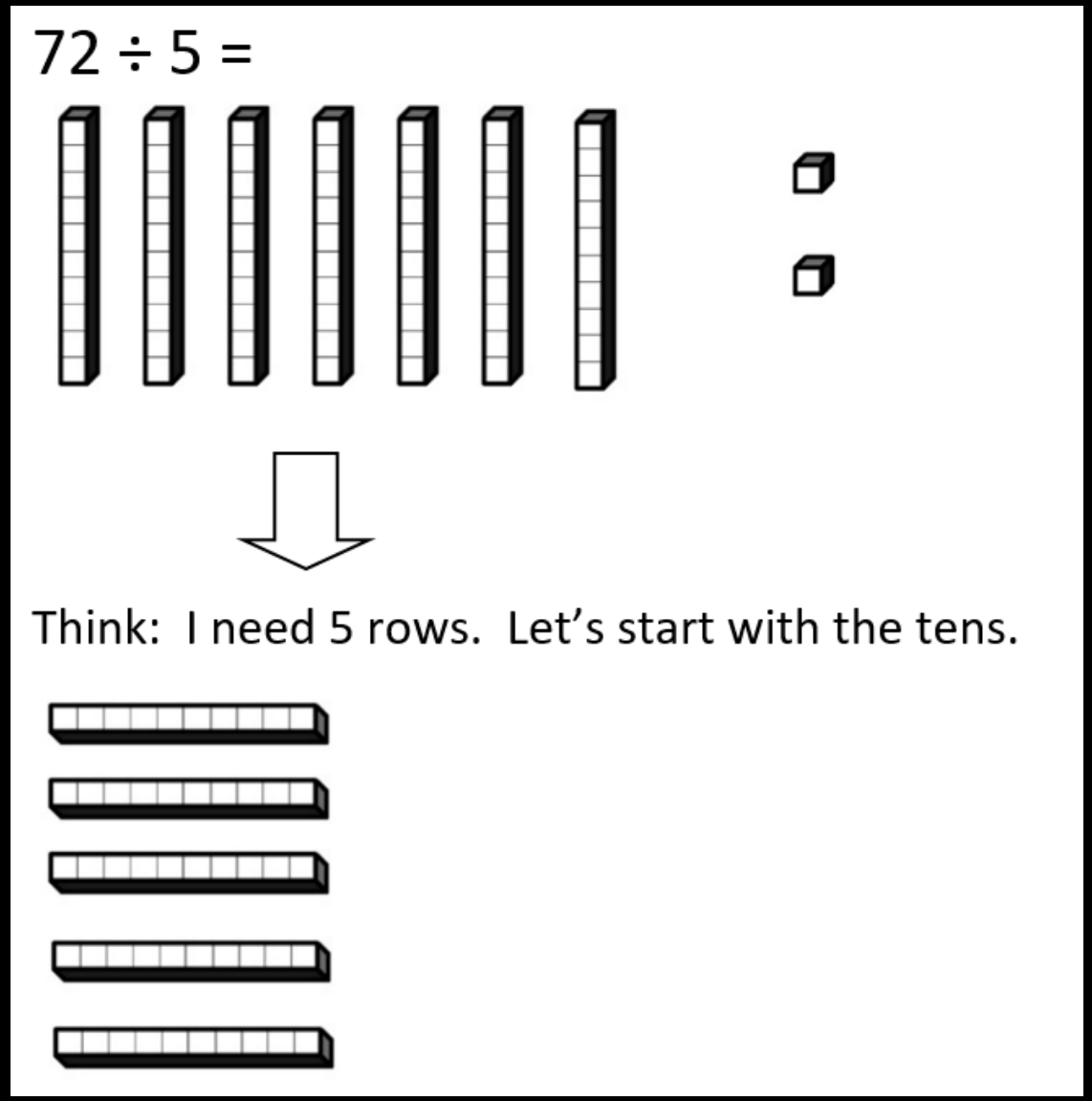
Division with remainders

72 ÷ 5

Think: what would this look like as an array?

How can I put 7 tens into 5 rows?

What will be left over?

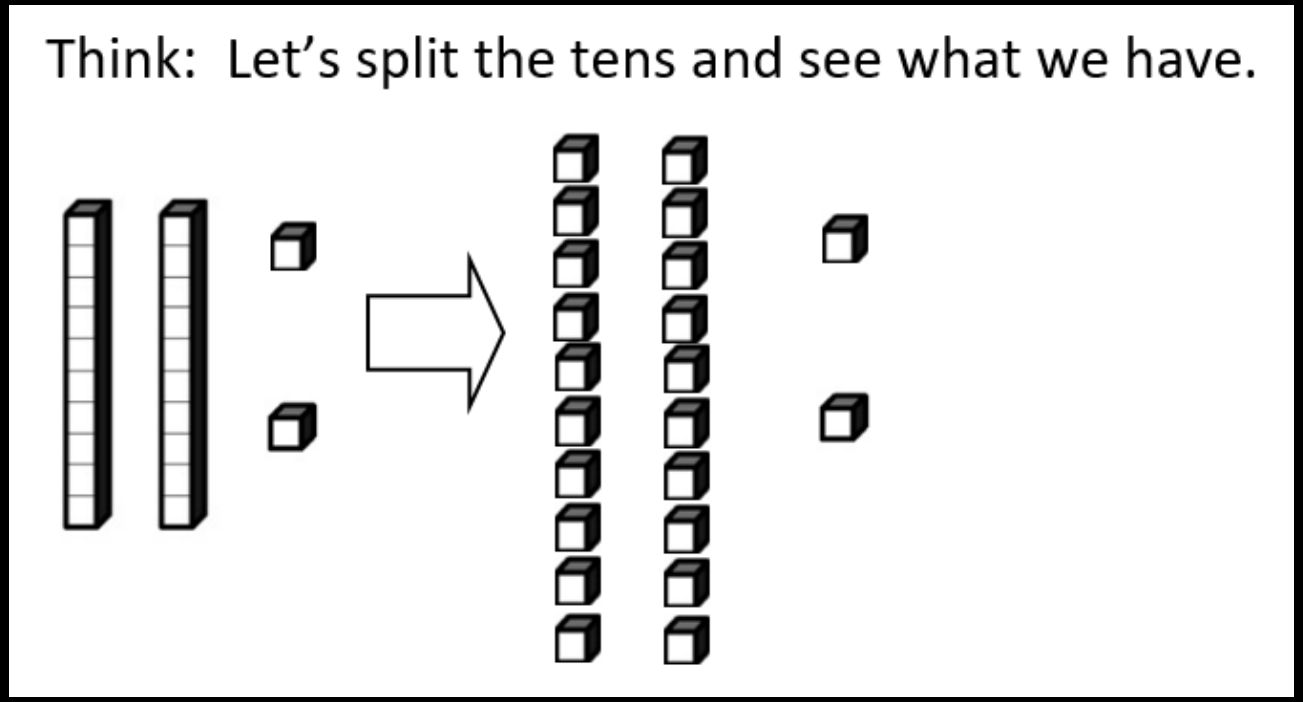


Division with remainders

72 ÷ 5

What should we do with the left over tens?

What do we need to do with the 22 ones?

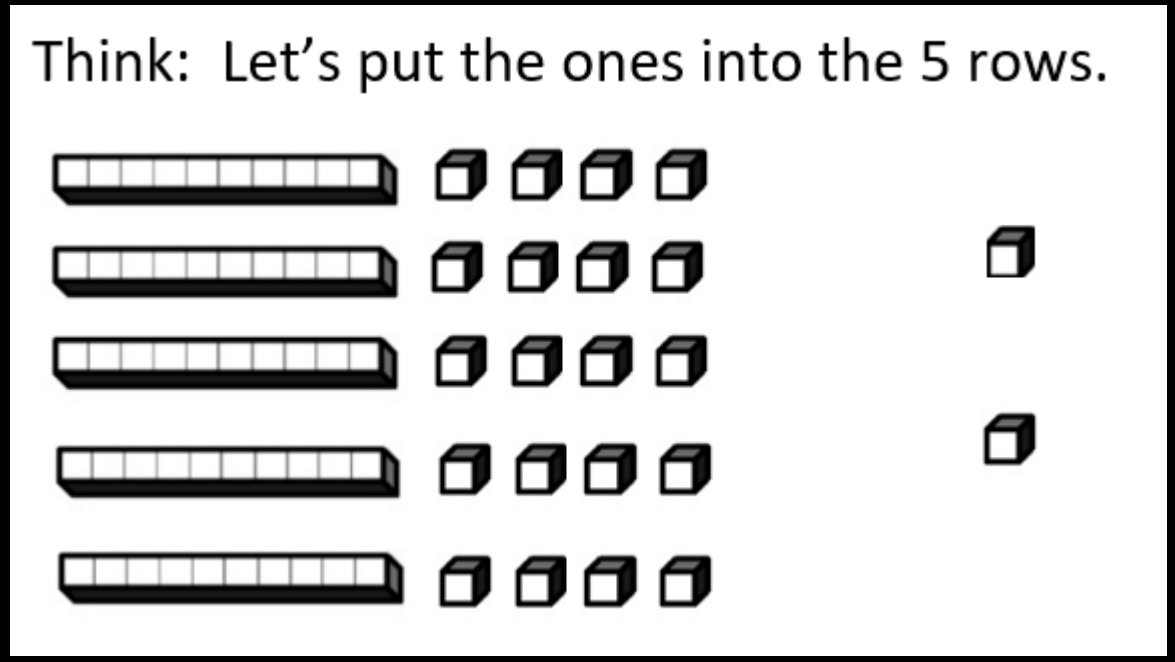


Division with remainders

72 ÷ 5

How can I put 22 ones into 5 rows?
What should we do with the left overs? What options do we have?

- Leave them (remainder)
- Cut them and spread out the bits

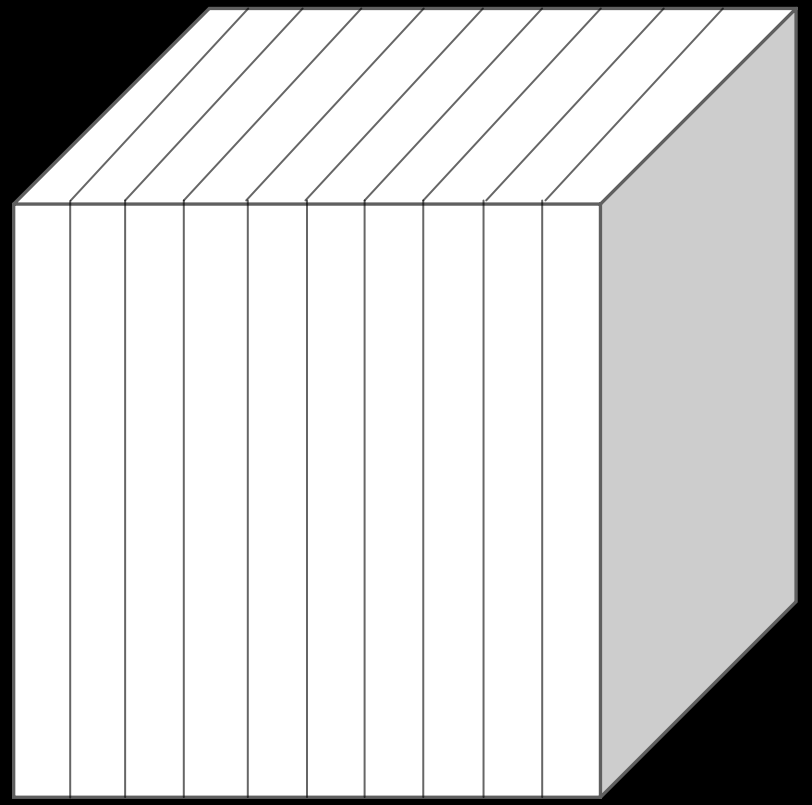


Linking division with fractions and decimals

72 ÷ 5

Think: How could we cut the 2 left overs to make five rows?

- Cut each one into fifths
- Cut each one into tenths



2 ones = 10 fifths

So 2 fifths are in each row

2 ones = 20 tenths

So 4 tenths are in each row

What is the same about 2/5 and 4/10?

Games and activities:

- How many rectangles can you make with 36 squares? Prisms?
- Shaker with 3 or 4 dice: Using any operation and any of the numbers once each, try to get as close as possible to…
- Array hunt or use lego
- Use dice to roll factors for arrays. Colour in the array on grid paper.
- Draw arrays of difficult to remember number facts. Partition the difficult number (e.g. 7 is 2 and 5), to break into easier parts (6 fives and 6 twos).
- If you know the perimeter, find the area.
- Find how many in a folded blanket, or covered grid.