Fixing misconceptions step 1: Diagnosing problems

NAPLAN and Misconceptions:

One widely-available source of non-routine questions that targets student misconceptions is NAPLAN tests. These questions are deliberately non-routine, yet with relatively simple content. The multiple choice questions even build common misconceptions into the possible answers!

Why routine questions don't cut it:

Kids don't want to risk being wrong or feeling stupid. Therefore, instead of answering a question with what they really believe or saying that they are not sure, students will tend to give the "right answers" whenever they can. If we only ask routine questions, then we will only get back routine answers. Unfortunately, these routine answers can hide underlying misconceptions which fit within the domain of understanding rather than memorisation.

If we want to diagnose misconceptions, we need to ask students questions to which they cannot have simply memorised the answers.

We need to ask non-routine or "weird" questions that deliberately target students' intuitive understanding of the underlying principles, patterns and connections in maths.

The lessons within this book contain diagnostic tasks that are designed to draw out student misconceptions. At times they will seem weird or even somewhat unfair. This is deliberate - we need to go beyond routine questions so that we can find what students really believe, rather than just what they have memorised.

How to ask diagnostic questions:

It is important to be aware that kids are very good at reading our body language. They often know than an answer is right or wrong just by our face and the tone of our voice!

When trying to diagnose misconceptions, we need to make sure that we are assessing a student's real understanding, not their ability to look at our eyebrows. Try to maintain a very neutral expression and tone of voice. If we express surprise or laugh at a wrong answer from a student they will know that it is wrong and will immediately censor their thoughts. If we don't know what they *really* think then we can't do anything to fix it. For tips on how to encourage risk taking in your class so that students are not concerned about being wrong, watch our short video at: http://goo.gl/PPLXq7

What to look out for:

When using diagnostic questions the purpose is to test the fragility of student understanding. Do they still understand the concept if we ask a weird question or does it seem to just fall out of their heads? Watch out for students who are watching your face closely to check if they are right. Watch for those who wait for others to answer first. Watch for those who answer, but with the tone of a question in their voice. Once you have spotted some kids in these categories check to see if they change their minds if you change your expression or voice.

Fixing misconceptions step 2: Confronting misconceptions

Why our current approaches aren’t working:

As teachers we are pretty good at knowing when a student doesn’t understand a concept. While we might not know exactly what is going wrong, diagnosing a problem is generally much easier than fixing it. This section shows how to confront and fix misconceptions so that the kids abandon that wrong idea and grow their intuitive understanding.

From what I have observed we tend to respond in one of these two ways when we find misconceptions:

1. Tell students that they are doing it wrong, and proceed to give them detailed steps to follow to get the answer right. We help them to practice the steps until they can "remember" them. The problem is that students are left trying to memorise steps that are in conflict with their intuitive understanding of maths because they haven't *changed their own mind* about the misconception first. They still have the underlying belief that is causing the problems because it hasn’t been undone.
2. Open up the problem by asking students to "try something else" - randomly guessing until they hit on the correct response and then trying to remember that one. Again, the misconception is still there because it hasn’t been undone.

Herein lies the difficulty,

Misconceptions are a problem with understanding, not with fluency.
Telling a student how to work out the answer and giving them steps to memorise does not fix understanding, it fixes fluency.

If kids fundamentally don't get a concept, they need to have this problem fixed before they can successfully learn a new concept. We need to undo their wrong understanding before we can lead them to develop a new concept. Otherwise we keep chasing our tails as our students keep "forgetting" what they have "learned".

How to use logical questions to confront and undo misconceptions:

To change our intuitive understanding or our beliefs about how something works we need to have a really good reason for doing so. Someone else telling us that we are wrong is generally not a good enough reason. We need to work out for ourselves that what we believe actually doesn’t make any sense and would never work before we are ready to abandon it.

To confront a misconception we need our students to figure out for themselves that what they are thinking doesn’t make any sense and would never work. We need to take them through a logical sequence of thinking that forces them to abandon their wrong ideas - but it needs to be their own thinking rather than ours. We can’t short cut the process by trying to substitute our thinking for theirs. That doesn’t work because they won’t have actually done the thinking for themselves and therefore won’t remember it afterwards.

To effectively confront misconceptions, we need to start by narrowing the options so that a student doesn’t have too much information to try and consider, and then make a series of very small changes to the situation until they realise that the situation itself doesn’t make any sense. By using a sequence of increasingly closed questions we limit the possible responses, presenting the student with situations that challenge and confront their thinking. These questions aim to point out the illogicality of a student’s thinking. They become more and more narrow, forcing the student to reconsider their initial answer each time until they realise this illogicality for themselves. A good example of this process is included in the next few pages.

Developmental sequence of concepts

Symbols:
DE – diagnostic-exploratory lesson (explained on page 9)
PB – pattern-building lesson (explained on page 10)
GA – generalising and application lesson (explained on page 10)

Lesson 1: Are different shaped halves the same size or not?

Lesson type: Diagnostic-exploratory in two parts Time Allocated: 1-1.5 hours

Concepts targeted:

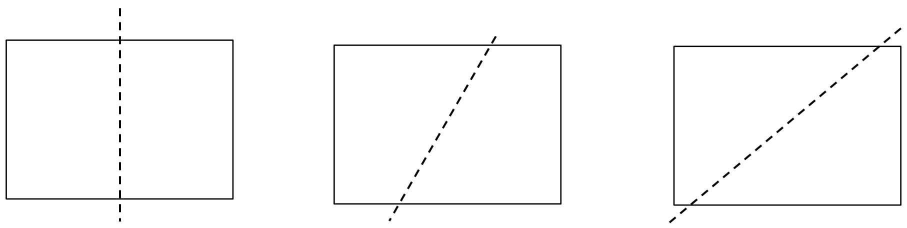
- Fractions need to be “fair”. If the pieces are to be given the same name, then the size of each piece needs to be the same.
- The “whole” needs to be the same when comparing fractions. We can’t add half of a triangle to one third of a circle – we need to start from the same “whole”.

Resources:

You will need lots of A4 paper, copies of the worksheet on page 24 and a pair of scissors for this activity.
To watch this lesson in action and with explanation, see the grade 4 lesson from the *Teaching Back to Front with Tierney* DVD series.

Step 1: Diagnostic question part A

Ask students to make half of an A4 piece of paper. Then ask them to make as many different halves as they can that are still really half of the paper. If they struggle, challenge them to make the “weirdest half” that they can which is still really a half. They should realise that there are limitless ways to fold a rectangle into halves as long as the fold line goes through the mid-point of the rectangle:



<u>Watch out for the following misconceptions:</u>

- Big halves and little halves
- Only symmetrical shapes can be half (e.g. you couldn’t fold the paper through the corners diagonally because everything wouldn’t line up nicely). These kids often cut the rectangle into a square and then cut the square diagonally, making three pieces.

Step 2: Confrontational questions for part A if needed

1. Cut along the fold and hold up both pieces. Ask the students if this was a really yummy cake which piece they would want to eat.

2. Ask if the pieces are halves. Ask why they think the pieces are halves or not halves. Hopefully this will establish the concept that halves have to be fair. Watch out for the term “even” as it is confusing to kids. “Fair” is a much better word to use than “even” as kids confuse the usage of the terms odd and even with that of equivalence.

3. Sometimes kids will tell you at this point that the pieces are halves because there are two of them rather than mentioning their size. If so, cut a really small corner off another piece and hand it to a student saying, “Ok then this can be your half and this can be my half. What do you think now?”. Usually the students will respond that the pieces aren’t fair. Ask if they think that matters or if they can still both be called halves if one is bigger.

4. Sometimes students decide that the pieces are not fair but that they are both still halves because half means two. I have actually had a student say, “I know it doesn’t make any sense Miss, but that is what they are called”. At this point try asking, “So we don’t think that these pieces are at all fair, but we still think that they should be called halves? Does that seem right to you?”. Usually students will respond that it doesn’t make much sense. I usually tell them that maths should make sense.

If something doesn't make sense, then it is usually wrong. Then I ask, "Does it make sense that things of totally different sizes would be given the same name, that both would be called halves?". Usually they say that it doesn't make sense. I say, "Well if it doesn't make sense, then it can't be right. These are going to go in my ***not halves*** category". I then create two categories on the board: halves and not halves. We test each idea for fairness and put each onto the board using one of those two categories.

5. For kids who cut the rectangle into a different shape first, get them to bring out all of the bits that they cut. Ask them to pick half. Then pick up the rest of the bits and say, "Ok you can have that half and I'll have the rest. What do you think?". Usually they realise that it isn't fair because they have cut off part of the paper first, then they go back to try again.

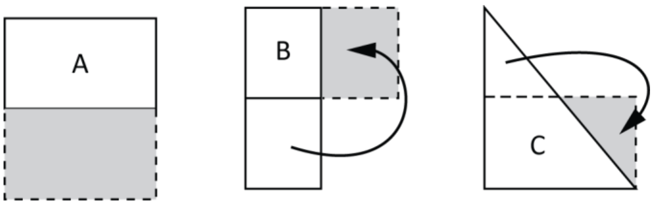
Step 1: Diagnostic question - part B (you can use the worksheet provided on page 24 for this question)
Label each of the differently shaped halves with a letter (A, B, C, D, E etc.) You will need to have at least four differently shaped halves, hopefully more. Ask the kids to write down the letter or letters for which of the pieces they think is the biggest. The pieces are actually the same size, but as this is a diagnostic question it is designed to show misconceptions. Typically, one of the students will ask if they can pick more than one shape. I usually respond with, "Sure – you can pick one, two, three, however many you want". I then check each student's answer individually by walking around and checking the letters that they have written. If they have answered with, "all of them" then I look at their faces – are they asking if that is right or are they really sure? If they aren't really sure then I double check their understanding by asking, "Ok, do you want a second guess in case you're wrong?" or, "Ok, but if they're not all the same then which one do you really think is the biggest?".

<u>Watch out for the following misconceptions:</u>

- Some kids believe that all halves have to be fair, but also believe that one or more of the halves you have identified are still bigger than the others. Students in this category are often missing *conservation of area* because they have not understood the two-dimensional array model of multiplication. An example of this thinking can be seen in the photograph on the right. This photograph shows 12 and 13 year old students voting that a particular half was bigger than the others even after they had gone through the initial testing phase of the first diagnostic question. This is a very persistent misconception, so make sure that you spend the necessary time to fix it properly using the questions below in step 2.

Step 2: Confrontational questions for part B

1. Hold up two of the halves in your hands. Ask the kids if there is a way that you could test to see if they really were the same size or if one was bigger. Typically, they will automatically try lining the shapes up and seeing if they can cut off the part that is overlapping and move it around to make both shapes the same (see below for process). If not, try holding the shapes together so that the dimensions align (place two sides of the same length together), then hold this up to the light so that the kids can see which parts line up and which parts don't. Usually this is enough to help them get the idea. Demonstrate cutting up the different shaped halves to overlay them if needed, making them all into the same shape as half A. Use this to check student answers.



2. Get the kids to check each shape and ask, "So which is bigger now?". Hopefully they will realise that each is actually the same size.

3. Double check that their understanding really has changed by moving the cut pieces back into their original position and asking, "So which is bigger now?". Hopefully they will realise that each is actually still the same size, but there are usually some kids who think that the size has changed again and one is bigger.

4. Repeat the process of moving the pieces, making tiny movements and asking, "How about now?" until the pieces are back into the same as shape A. Usually the response is, "bigger, bigger, bigger… now they are the same". Make a tiny movement to shift the two pieces apart again and ask, "How about now?". See if the student thinks that it is bigger/smaller again or if they are still the same. Hopefully just making tiny movements to isolate the spot where the shape "becomes bigger" will be enough to change their minds. If not, move on to step 5.

5. Cut shape A to be the same as shape B. Now move the pieces of shape A to look like shape B, keeping shape B's pieces in the position of shape A. Ask again, "How about now?". Often at this point a student says, "Now this is confusing – I don't know which one is bigger!". If so, you can respond with, "Well, is one actually bigger or does it just look bigger? Which one has more cake if you could eat both pieces?". If not, try step 6.

6. Swap one of the pieces of shape A with an identical piece from shape B. Ask again, "How about now?". Keep swapping the pieces and moving them back into the other positions until the student responds by saying either that they are the same or that they don't know. Question as per step 5 above. If they don't do this, you will need to go back to an intervention on drawing arrays for multiplication facts (e.g. draw 3 x 4 as three rows of four, then rotate the drawing and see if it is still the same). Please note: As this is a separate concept from fractions it has not been included within this program. Information and lessons on how to fix multiplicative thinking can be found on the Back-to-Front Maths website. A videoed lesson showing how this misconception was fixed in a grade six class can also be found in the DVD series, *Teaching Back to Front with Tierney*.

Step 3: Leading questions

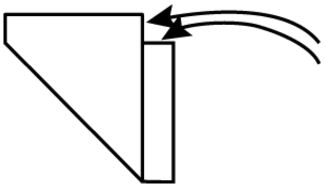
- "Do you think I can have a big half and a small half or do they have to be **fair**?".
- "Is it a different amount of cake, or does it just look different? Can it look different but still be **fair**?".
- "What have you changed your mind about today? What have you learned?".

Step 4: Generalising questions:

- "Do you think that halves always have to be fair? So, what if they look different but are the same size really - would that still be ok?".
- "Ok, let's try making the weirdest shaped half that we can from this A4 paper, but it still has to be fair".
- "How about for other fractions? Do they have to be fair too or is it just for halves?".
- Reiterate: "So, we have learned today that fractions have to be fair".

Differentiation:

Extension students: Have them work out other weird ways to fold the paper to still make half. One interesting principle that a *grade one* student came up with is this:



As long as these two distances are the same it will always be one half!

Support students:

1. Reinforce the concept by cutting halves from other shapes.
2. Draw arrays of simple multiplication facts (e.g. 2 x 3) and then swivel them by 90° to see that it is the same amount. Repeat with lots of different grids or arrays (e.g. 6 x 5 = 5 x 6), then go back to fractions.