What to do when kids already know everything - serious maths extension

By Tierney Kennedy
QAMT executive counsellor
Author of *Back-to-Front Maths*

Sometimes when teaching, we come across a student who blows our mind. It seems like they either already know everything that we are trying to teach, or are so fast at making connections that we can't keep up. In this situation it can be really hard to know what to, especially if the student seems to understand more maths than we are comfortable with. This article provides four simple keys for extending student thinking and skills, without just making the numbers harder.

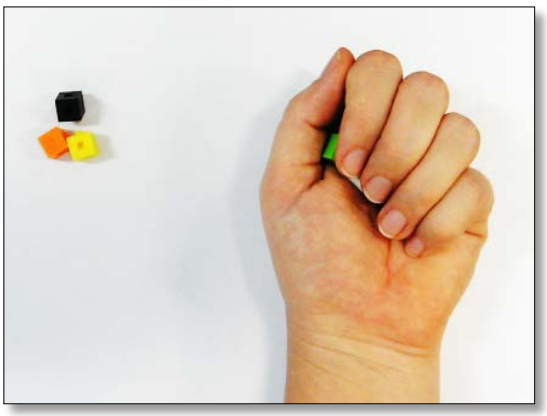
Key 1: Make the questions weirder rather than making the numbers bigger

Students with high mathematical understanding tend to find routine-style questions very easy and often end up bored and frustrated. At times we are tempted to give "fast finishers" more and more routine questions to fill in time, effectively punishing them for having higher understanding than we were prepared for. In contrast to this approach, the Australian Curriculum recommends that students "make connections between related concepts and progressively apply the familiar to develop new ideas" (Australian curriculum website, 2015). To develop strong understanding in students we therefore need to consider how to use unfamiliar or unusual tasks to challenge their thinking.

One way to do this is to change the structure of routine questions rather than simply making the content harder. Here are a few ideas which can be easily prepared before-hand and adapted to any content area:

1. Think about using gaps, missing add-ends, or asking students to work backwards to a starting number rather than simply working forwards to an unknown answer. The picture below shows what happened when a teacher started with eight blocks and hid some.
   $\square + 3 = 8$ and $3 + \square = 8$ are both much harder for kids to solve than $3 + 5 = \square$

2. Give students the starting and ending numbers and ask them what happened to get that answer – get them to work out what operation/s were used.



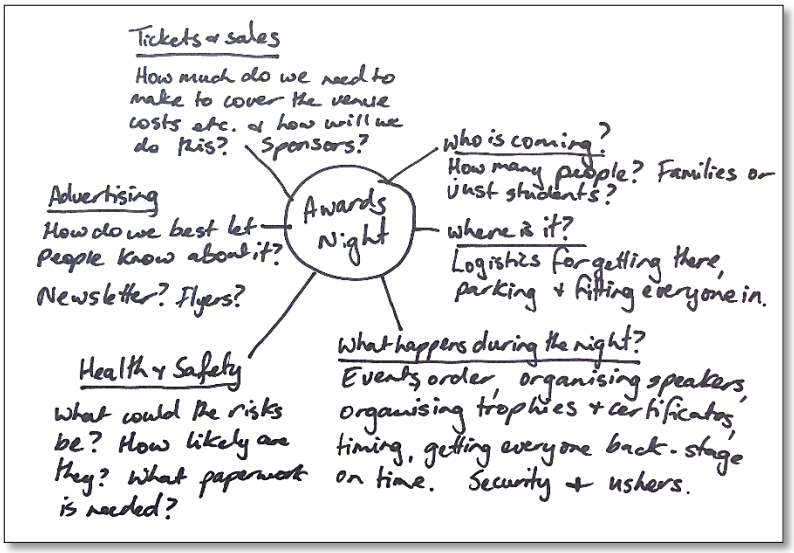
3. Require students to adapt what they have already worked out to account for a change in circumstance or additional operations, for example: *What if I had also subtracted two to get the answer of eight? How would that change the number in the box?*
   How would your answer change if the three was a different number?

4. Add in an additional unknown using a pattern, for example: *What if we had three numbers that added to give eight, but two of the numbers were the same?*

5. Use a non-standard looking question, for example: present fractions in a 3D solid rather than an area model, use irregular rather than regular shapes, ask students to work out the time on an analogue clock without the minute hand.

Key 2: Make use of real-life events happening at your school

Consider what events you have coming up this term at school. Do you have an athletics carnival, concert, awards night, disco, swimming carnival, art show or excursion? These are fantastic sources of real-world mathematical thinking that are readily engaging for students. In order to make the best use of these situations, I find it is best to start out by thinking of what jobs/tasks need to be done to make the event successful rather than starting out by thinking about what mathematics is involved in the task. Check out the example below for organising an awards night:



Once you have teased out the task, choose one aspect and apply your "maths teacher" hat. What content areas best lend themselves to this job? What would be suitable for your students? What would they enjoy doing? For example: you might get students to organise the schedule for the night, including what time everyone needs to be backstage and ready to go on, when the lights/curtains/speakers/presentations happen etc. Another idea would be for students to work out the logistics and recommend a venue (how many people are we expecting and therefore how many seats/toilets/exits/drinks/tickets… are needed?), or work out how to get everyone from school to the venue for the practice run.

Key 3: Make them prove it

I find that often students with high mathematical understanding have difficulties explaining what they did to find the answer to a question. I expect that this is because they already knew the answer, or saw the solution so quickly that their thinking did not slow down enough to be able to identify the steps involved. Mathematical reasoning and communication are critical for these students to develop if they are to share their ideas, proofs or solutions with anyone beyond school. If we want students to explain their thinking, there are a number of strategies worth trying:

1. If students can do it in their heads, then the question is probably not hard enough. Make the maths harder so that students can't just see the answer by adding in enough different ideas, connections and steps that they need to write something down to keep track of everything that they are doing.

2. Instead of asking the student to explain what steps they went through, ask them to prove why a different answer is wrong. Personally, I've found that I get the best reasoning from students who are trying to prove that my answer is wrong!

3. Give the students a number of questions that all follow a similar structure and then ask them to explain the similarities and differences, as well as explaining how the questions are connected.

Key 4: Create an extension box

While it is necessary to include extension students in many of the normal activities that you do with your class, consider which lessons or parts of lessons they could use to do something more challenging. One great idea is to create a box in your classroom in which you place a challenging task for two to three students to work on together. This box can be accessed whenever you feel that it will be useful, for example: when students have finished their work, when you are explaining lower-level concepts, when the class is practicing skills that these students have already mastered. Each task should be written in the form of a student brief, with a simple description of what is required along with the key steps and/or requirements.

Here are a few simple examples from the *Back-to-Front Maths* website:

- You have been given a budget of $50. You need to plan a meal for ten people, make a shopping list, and work out the total cost for the meal. You will be provided with a recipe book and a catalogue from a local grocery store.
- Your school wants to apply for federal funding to build a new playground. As part of the grant application process the administration has decided to take suggestions from students on the design of the playground. Your task is to design a playground that will fit in the area of your school that your teacher indicates. You must include a map of your playground which shows a variety of shapes such as quadrilaterals and triangles on the ground. These can be garden beds, paths, or sections around playground equipment. Your map must include the mapping conventions. A second map needs to contain details about the shapes within your playground (angles, sides). You must also hand in a 3D model of a climbing frame that you design.
- School planners and architects need to create models of rooms so that they can design spaces that work well. Your job is to use lego or other blocks to create a model of your classroom, including where the furniture is and where the door is. If you use lego, try to use 4 lego dots for each metre in length, but don't worry if it is a bit over or a bit under.

Whatever you do, remember that maths should be fun, challenging and should make kids think hard. Solving a new problem, adapting to a new situation, trying to find a pattern and modelling a real-life situation are a lot more engaging than answering routine questions. And that's true for all students, not just those at the top-end.